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AEC-NASA SPACE NUCLEAR SYSTEMS OFFICE

NERVA RELIABILITY AND SAFETY ANALYSIS METHODS

VOLUME IV - RELIABILITY TESTING, TREND DATA & FAILURE ANALYSIS

NERVA PROGRAM, CONTRACT SNP-1

JUNE 1972

AEROJET NUCLEAR SYSTEMS COMPANY

A DIVISION OF AEROJET-GENERAL

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APPROVED BY:

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NERVA PROGRAM RELIABILITY PROCEDURE	NUMBER: R101-NRP-501	REVISION B
	EFFECTIVE DATE: 1 June 1972	CATEGORY III
RELIABILITY REVIEW OF TEST PLANS	SUPERSEDES: Reliability Review of Test Plans NUMBER: R101-NRP-501 DATE: 1 June 1971	
	APPROVED BY: W. M Bryan	

ABSTRACT

This procedure establishes guidelines for Reliability review of test plans and/or specifications for materials, major components, subsystems, and systems during development, prequalification, qualification, and flight testing of the NERVA engine. The procedure establishes a uniform review process for all test plans/specifications to assure that: (1) critical failure modes and failure mechanisms, identified by component and system failure mode analysis, are properly investigated by appropriate testing, (2) statistical experimental design techniques are employed where appropriate, (3) test data generated is sufficient to satisfy design, reliability, and trend evaluation objectives, (4) an integrated test plan concept is followed, and (5) reliability considerations are an integral part of testing and qualification phases as set forth by the Reliability Program Plan, R-101, for the NERVA Program.

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1.4 Data Item R-202, Reliability Allocations, Assessments, and Analysis Report

1.5 Data Item T-101, System Test Plan

1.6 Data Item T-102, Test Plans/Procedures (when approved plan is available)

2.0 POLICY AND RESPONSIBILITY

2.1 POLICY

2.1.1 Test plan review shall be conducted by Reliability to determine and assure the inclusion and adequacy of provisions in test plans to satisfy test objectives, Reliability Program Plan (Data Item R-101) requirements, and Reliability and Flight Safety Evaluation (Data Item R-106) requirements.

Reliability reviews and participation shall assure that statistical design of experiment techniques are used throughout all test planning as appropriate. This review and participation will be performed during the development of test plans to avoid "after the fact" reviews.

2.1.2 An integrated test plan concept shall be followed wherever appropriate. Under this concept, the test objectives of any specific test plan shall be in concurrence with the overall test program objectives.

2.1.3 The final draft of test plans/specifications shall be approved by Reliability prior to implementation.

2.2 APPLICABILITY

This procedure applies to the Reliability review of all Development, Prequalification, and Qualification test plans/specifications prepared for each of the components and assemblies identified as Contract Prime (CP), Engineering Critical (EC), or Design Sheet (DS) in the NERVA Engine Specification Tree and their associated materials test plans. Informal review will also be made of all Exploratory Development test plans.

2.3 RESPONSIBILITIES

As specified in M-001, the NERVA Management Plan, Reliability will participate in the review/approval cycle of test plans/specifications identified by this procedure. The ANSC NERVA Reliability Manager will approve all test plans for tests identified h the nonnuclear subsystem. The WANL NERVA Reliability Manager will approve those identified with .e nuclear subsystem.

3.0 PROCEDURE

3.1 ACQUISITION

As specified in M-001, the NERVA Management Plan, each test plan document (prepared in accordance with program directives) shall be forwarded to Reliability by the activity initiating the plan. This may be a draft copy distributed to all activities for review and comment. The review copy should normally be forwarded three days before comments are due. In addition to this formal review and approval, Reliability shall be notified early in the formation phase of each test plan so that recommendations can be easily incorporated.

3.2 TEST PLAN REVIEW

Reliability shall review test plans for the following:

3.2.1 Adequacy of Test Objectives

In general, the test objectives should specify in terms of primary and secondary purposes what must be learned from the testing, which questions must be answered, and what effects on parameters must be estimated. The test plan should also be identified as to type, e.g., preliminary, exploratory, developmental, final demonstration, reliability assessment, etc. It should also be clear how the specific test plan relates to the overall test program.

3.2.2 Adequacy to Evaluate Failure Modes and Design Weaknesses

Are specific failure modes/mechanisms as identified by the modes of failure analysis properly considered in the test plan? Are the critical failure modes and/or associated margins of safety properly investigated?

3.2.3 Suitability of Test Conditions

Do the levels of the imposed test environments agree with the environmental levels anticipated during engine usage as specified in paragraph 1.1? Are combined environments imposed during testing, if feasible? Do the imposed test environments match all usage environments as specified in paragraph 1.1?

3.2.4 Adequacy of Proposed Test Times/Cycles

Do the proposed test times/cycles reflect the anticipated usage test times/cycles as specified in paragraph 1.1? Has proper consideration been given for testing to failure?

3.2.5 Adequacy of Test Response Variables

Is provision made for monitoring the proper test response variables? Table 1 presents, as a guide, a list of typical failures experienced as a result of application of specified environments. Is provision made for evaluating the typical induced failures presented in Table 1?

3.2.6 Proper Definition of Failure and/or Success Criteria

Are the minimum/maximum parameter limits, as defined by component/engine specifications, delineated in the test plan?

3.2.7 Proper Use of Statistical Experimental Design Techniques

Has the test program been statistically designed? Are there provisions for estimating experimental error? Has randomization been considered in the selection of test items and in the implementation of the testing sequence? Can possible interactions be evaluated?

3.2.8 Susceptibility to Statistical Analysis of Resultant Test Data

Is the resultant test data conducive to analysis by analysis of variance techniques? Will the trend data requirements be satisfied?

3.2.9 Physical Limitation

Does the proposed testing procedure allow tests to be conducted as specified, considering limitations which could be imposed by the test equipment/facilities, test operators, and test scheduling?

3.2.10 Adequacy of Data Collection, Logging and Reporting

Are the data collection, logging and reporting requirements adequate and completely defined? Is there provision for recording unusual or unforeseen occurrences?

3.2.11 Precision and Accuracy Requirements vs Capability

Are the required accuracy and precision of the test equipment specified? Does the test equipment employ instrumentation, sensors, and recording devices with accuracy and precision limits within the expected variation of the test variables specified?

3.2.12 Reliability Data Requirements

Will the test program produce the data required for reliability predictions/assessments as specified in Data Item R-106 and required for Data Item R-202?

3.2.13 Adequacy of Test Plan

Will the test plan, as presented, satisfy the test objectives? Are a sufficient number of tests planned to provide the required precision on estimates or planned comparisons? Section 3.3 discusses methods for verifying the number of tests.

3.2.14 Integrated Test Plan Concept

Does the proposed test plan consider the relation of the test being conducted with other interacting component and system tests being considered? Is the test plan objective in concurrence with the overall test program objectives? Does the test plan define the criteria of failure of this test? For preliminary or exploratory type testing, an integrated test plan concept may not be appropriate; however, for development, prequalification, qualification, flight assurance and flight rating testing, the concept shall be followed.

3.3 VERIFICATION OF SAMPLE SIZE

This section presents techniques by which Reliability will judge the adequacy of a test plan with respect to sample size. Such an assessment requires comparison of the plan against specified criteria dependent upon the specific objectives of the test program. Four basic types of objectives are covered. It is recognized that not all test objectives will fit in these categories and slight modifications in the techniques may be required in some cases.

3.3.1 Required Information

In general it is necessary to know the expected variation in the test data and the degree of difference it is important for the experimenter to detect. The specific information required for each of four types of test objectives is specified below. Standard assumptions are supplied for those cases in which the test planner is unable to provide the necessary data.

3.3.1.1 Tests for Differences Between Means

What size of difference $(\mu_x-\mu_y)$ is it important to detect? What is the expected experimental error, σ (standard deviation from identical tests)? Information may be supplied in the form $(\mu_x-\mu_y)/\sigma$. If no logical basis is yet available to provide this information a maximum value of $(\mu_x-\mu_y)\sigma = 2.0$ will be used.

3.3.1.2 Tests to Determine If a Correlation Exists

What size change in y (dependent variable) is it important to detect for a specified change in x (independent variable)? What is the expected standard deviation, σ, for y at a fixed x. Information may be supplied in the form $\Delta y/\sigma$ versus Δx. If information is not available a maximum value of $\Delta y/\sigma = 2.0$ will be assumed with $\Delta x = x_{max} - x_{min}$, the difference between the maximum and minimum levels of x used in the test plan.

3.3.1.3 Tests to Estimate a Mean

How close is it important for the estimated mean to be to the true mean and what is the expected variation, σ? This information may be given in the form $|\hat{\mu}-\mu|/\sigma$ where $\hat{\mu}$ is the estimated mean and μ is the unknown true mean. If no logical basis is yet available to provide this information a maximum value of $|\hat{\mu}-\mu|/\sigma = 1.5$ will be used.

3.3.1.4 Tests to Estimate a Regression Line When it is Known a Correlation Exists

How close is it important for the estimated mean (line) to be to the true mean (line) for what specific value or between what limits on x, the independent variable? What is the expected standard deviation, σ, for y at a fixed x. The information may be given in the form $|\hat{\mu}(x)-\mu(x)|/\sigma$ for any x or range of x's. If necessary a maximum value of $|\hat{\mu}(x)-\mu(x)|/\sigma = 1.5$ will be assumed at the extreme levels of x used in the experiment.

3.3.2 Evaluation Procedures

The following procedures are based on the probabilities of Type 1 and Type II errors being controlled to $\alpha \leq .05$ (one sided) and $\beta \leq .10$ respectively. (In a statistical test of hypothesis a Type I error is committed if the hypothesis is rejected when in fact it is true, and a Type II error is committed if the hypothesis is accepted when in fact it is false). In each case, f denotes the degrees of freedom associated with the appropriate error term and n_i the number of tests planned under the i^{th} condition. For te : plans of a complex nature a statistician should be consulted to ensure proper determination of the degrees of freedom.

3.3.2.1 Tests for Differences Between Means

Step 1. Determine f

Step 2. Determine n_1 and n_2, the two smallest sample sizes

Step 3. Obtain δ from Table 2.

Step 4. Calculate $C = \delta\sqrt{\frac{n_1+n_2}{n_1 n_2}} = \delta\sqrt{\frac{2}{n}}$ for $n_1 = n_2 = n$

Step 5. If $C > (\mu_x-\mu_y)/\sigma$ reject plan

$< (\mu_x-\mu_y)/\sigma$ accept plan

3.3.2.2 Tests to Determine if a Correlation Exists

Step 1. Determine f

Step 2. Calculate $S_{xx} = \Sigma n_i (x_i - \bar{x})^2$

Step 3. Obtain δ from Table 2.

Step 4. Calculate $C = \delta \Delta x / \sqrt{S_{xx}}$

Step 5. If $C > \Delta y/\sigma$ reject plan

$\leq \Delta y/\sigma$ accept plan

3.3.2.3 Tests to Estimate a Mean

Step 1. Determine n and f

Step 2. Obtain $t_{.05,f}$ (one-sided) from standard t tables

Step 3. Obtain $\chi^2_{.10,f}$ from standard chi-square tables (10% upper tail)

Step 4. Calculate

$$C = \frac{nf}{t^2_{.05,f}} \left(\frac{\hat{\mu} - \mu}{\sigma}\right)^2$$

Step 5. If $C < \chi^2_{.10,f}$ reject plan

$\geq \chi^2_{.10,f}$ accept plan

3.3.2.4 Tests to Estimate a Regression Line When it is Known a Correlation Exists

Step 1. Determine f

Step 2. Calculate $\bar{x} = \Sigma n_i x_i / \Sigma n_i$

Step 3. Select x_o such that $|x_o - \bar{x}|$ is maximum for the required x range

Step 4. Calculate $S_{xx} = \Sigma n_i (x_i - \bar{x})^2$

Step 5. Calculate effective $n = 1/[\frac{1}{\Sigma n_i} + \frac{(x_o - \bar{x})^2}{S_{xx}}]$

Step 6. Obtain $t_{.05,f}$ (one-sided) from standard t tables

Step 7. Obtain $\chi^2_{.10,f}$ from standard chi-square tables (10% upper tail)

Step 8. Calculate $C = \frac{nf}{t^2_{.05,f}} \left(\frac{\hat{\mu}(x) - \mu(x)}{\sigma}\right)^2$

Step 9. If $C < \chi^2_{.10,f}$ reject plan

$\geq \chi^2_{.10,f}$ accept plan

3.3.3 <u>Example Evaluations</u>

3.3.3.1 Tests for Difference Between Means

Plan A 4 tests on each of 5 conditions

$f = 5(4-1) = 15 \quad n_1 = n_2 = n = 4$

From Table 2, $\delta = 3.069$

$C = \delta\sqrt{\frac{2}{n}} = 3.069\sqrt{\frac{2}{4}} = 2.17 > 2.0$ reject plan

Plan B 5 tests on each of 3 conditions

$f = 3(5-1) = 12 \quad n_1 = n_2 = n = 5$

From Table 2, $\delta = 3.109$

$C = \delta\sqrt{\frac{2}{n}} = 3.109\sqrt{\frac{2}{5}} = 1.97 < 2.0$ accept plan

3.3.3.2 Tests to Determine if a Correlation Exists

Plan A

Stress, x	No. Tests
20	4
18	4
16	4

$f = 12-2 = 10$

$S_{xx} = 4\left\{(20-18)^2 + (18-18)^2 + (16-18)^2\right\} = 32$

Assume $\Delta x = 20-16 = 4$

From Table 2 $\delta = 3.149$

$C = \delta\frac{\Delta x}{\sqrt{S_{xx}}} = \frac{(3.149)(4)}{\sqrt{32}} = 2.23 > 2.0$ reject plan

Plan B

Stress	No. Tests
20	4
19	4
18	4
17	4
16	4

$f = 20-2 = 18$

$S_{xx} = 4\left\{(20-18)^2 + (19-18)^2 + (18-18)^2 + (17-18)^2 + (16-18)^2\right\}$

$= 40$

Assume $\Delta x = 20-16 = 4$

From Table 2 $\delta = 3.044$

$$C = \delta \frac{\Delta x}{\sqrt{S_{xx}}} = \frac{3.044 \times 4}{\sqrt{40}} \approx 1.93 < 2.0 \text{ accept plan}$$

3.3.3.3 Tests to Estimate a Mean

<u>Plan A</u>

4 tests to be conducted to determine mean. Wants estimate to be within 15% of true mean. Experience indicates $\sigma \approx 10\%$ of mean

$n = 4 \qquad f = n-1 = 3$

From t- tables $t_{.05,3} = 2.353$

$$C = \frac{nf}{t^2_{.05,3}} \left(\frac{\Delta}{\sigma}\right)^2 = \frac{(4)\ (3)}{(2.353)^2} \left(\frac{.15}{.10}\right)^2$$

$$= 4.87 < 6.25 = \chi^2_{.10,\ 3} \quad \text{reject plan}$$

A re-evaluation with n = 5 provides an acceptable plan.

3.3.3.4 Tests to Estimate a Regression Line

<u>Plan A</u>

Stress, x	No. of Tests
20	8
18	8
16	8

Desires estimate of mean for all stresses from 16 to 20 to be within 5%. Expected variation for a fixed stress is 10%.

$f = 3(8)-2 = 22 \qquad \bar{x} = 18 \qquad x_o = 16 \text{ or } 20$

$$S_{xx} = 8 \left\{(20-18)^2 + (18-18)^2 + (16-18)^2\right\} = 64$$

$$n = 1/\left[\frac{1}{\Sigma n_i} + \frac{(x_o-\bar{x})^2}{S_{xx}}\right] = 1/\left[\frac{1}{24} + \frac{(16-18)^2}{64}\right] \approx 9.6$$

$$t_{.05,f} = 1.717 \qquad \frac{|\hat{\mu}(x) - \mu(x)|}{\sigma} = \frac{.05}{.10} = .5$$

$$C = \frac{nf}{t^2_{.05,f}} \left(\frac{\hat{\mu}(x) - \mu(x)}{\sigma}\right)^2 = \frac{(9.6)\ (22)}{(1.717)^2} (.5)^2$$

$$= 17.9 < 30.8 = \chi^2_{.10,22} \text{ reject plan}$$

NOTE:

$$\text{At } x_o = 17 \qquad n = 1/[\frac{1}{24} + \frac{(17-18)^2}{64}] = 17.5$$

$$C = \frac{(17.5)\ (22)}{(1.717)^2} (.5)^2 = 32.6 > 30.8$$

So plan gives desired precision from stresses of 17 to 19.

3.4 REVIEW DOCUMENTATION

Reliability will forward (by written memo) the results of the review to the originating activity. Reliability will provide follow-up of review comments and will provide any appropriate assistance to the author in incorporating the comments.

3.5 REVIEW STATUS LOG

Reliability shall maintain a log of all test plans and specifications reviewed. The log shall include:

3.5.1 Test plan/specification identification.

3.5.2 Date received, date reviewed, and the date review comments were forwarded to originating activity.

3.5.3 Person responsible for review.

3.5.4 Brief summary of significant comments.

3.5.5 Summary of required followup activities.

TABLE 1 - ENVIRONMENTAL EFFECTS AND FAILURES INDUCED NRP 501A

ENVIRONMENT	PRINCIPAL EFFECTS	TYPICAL FAILURES INDUCED
Acceleration	Mechanical stress	Structural failure
Dissociated gases	Dielectric strength reduced; Chemical reactions; Contamination	Alteration of Electrical Properties; Alteration of Physical Properties; Insulation breakdown or arcover
Explosive decompression	Gross mechanical stress	Structural failure; Rupture or cracking
High pressure	Compression	Structural failure; Penetration of seals; Interference with function
High relative humidity	Corrosion; Electrolysis; Moisture absorption	Loss of mechanical strength; Loss of electrical strength; Increased conductivity of insulation; Physical breakdown; Swelling; Interference with function
High temperature	Physical expansion;	Increased wear on moving parts; Structural failure; Alteration of electrical properties; Insulation failure; Loss of lubrication properties
Low pressure	Outgassing; Expansion; Reduced dielectric strength of air	Alteration of electrical properties; Explosive expansion; Structural failure; Insulation breakdown and arcover Corona and ozone formation
Low relative humidity	Desiccation (embrittlement or granulation)	Structural failure; Loss of mechanical strength; Alteration of electrical properties
Low temperature	Physical contraction; Embrittlement; Increased viscosity and solidification	Structural failure; Cracking or fracturing; Loss of mechanical strength; Alteration of electrical properties; Loss of lubrication properties; Increased wear on moving parts

TABLE 1 (Cont) NRP 501A

ENVIRONMENT	PRINCIPAL EFFECTS	TYPICAL FAILURES INDUCED
Magnetic fields	Induced magnetization	Alteration of electrical properties; Induced heating Interference with function
Mechanical impact shock	Mechanical stress	Structural failure
Nuclear irradiation	Heating; Transmutation and ionization	Thermal aging; Oxidation; Alteration of physical, electrical and chemical properties; Production of gases and secondary nuclear particles
Ozone	Chemical reactions; Reduced dielectric strength of air	Rapid oxidation; Loss of mechanical strength; Interference with function; Alteration of electrical properties; Insulation breakdown and arcover
Rain	Physical stress; Water absorption and immersion; Erosion; Corrosion	Structural failure; Increase in weight; Aid heat removal; Electrical failure; Surface deterioration; Enhances chemical reactions
Sand and Dust	Abrasion; Clogging	Increased wear; Alteration of electrical properties; Interference with function
Salt Spray	Corrosion; Electrolysis	Increased wear; Loss of mechanical strength; Structural failure or weakening; Interference with function; Alteration of electrical properties; Increased conductivity;
Solar radiation	Actinic and Physico-chemical reactions	Surface deterioration; Alteration of electrical properties; Discoloration of materials; Ozone formation

TABLE 1 (Cont) NRP 501A

ENVIRONMENT	PRINCIPAL EFFECTS	TYPICAL FAILURES INDUCED
Temperature shock	Mechanical stress	Structural failure or weakening; Seal damage
Vibration	Mechanical stress; Fatigue	Structural collapse; Loss of mechanical strength; Increased wear; Interference with function
Wind	Force application; Deposition of materials; Heat loss (low velocity) Heat gain (high velocity)	Structural collapse; Loss of mechanical interference and clogging; Abrasion accelerated; Accelerates low-temperature effects; Accelerates high-temperature effects
Zero gravity	Mechanical stress;	Interruption of gravity-dependent function; Aggravation of high-temperature effects

TABLE 2

VALUES OF THE NON-CENTRALITY

PARAMETER, δ, FOR A ONE-SIDED TEST WITH α = .05, β = .10

f	δ	f	δ	f	δ
1	10.51465	21	3.02610	85	2.95002
2	4.80923	22	3.02132	90	2.94868
3	3.92820	23	3.01698	95	2.94750
4	3.59995	24	3.01302	100	2.94643
5	3.43174	25	3.00940	110	2.94459
6	3.33014	26	3.00606	120	2.94306
7	3.26231	27	3.00298	130	2.94177
8	3.21389	28	3.00013	140	2.94066
9	3.17761	29	2.99749	150	2.93971
10	3.14944	30	2.99502	160	2.93886
11	3.12692	35	2.98488	170	2.93813
12	3.10854	40	2.97736	180	2.93747
13	3.09322	45	2.97155	190	2.93688
14	3.08029	50	2.966[illegible]	200	2.93636
15	3.06920	55	2.90316	∞	2.92611
16	3.05961	60	2.90005		
17	3.05[illegible]23	65	2.95741		
18	3.01382	70	2.95515		
19	3.03726	75	2.95322		
20	3.03139	80	2.95151		

NERVA PROGRAM RELIABILITY PROCEDURE	NUMBER: R101-NRP-502	REVISION
	EFFECTIVE DATE:	CATEGORY III
SAMPLING FOR FATIGUE TEST	SUPERSEDES: NUMBER: DATE:	
	APPROVED BY: *G R Renstifter*	

1.0 PURPOSE

Fatigue life is one of the important mechanical properties of components/parts subjected to cycling loads. To estimate fatigue life, specimens of the parent material are tested to failure or for large numbers of cycles to determine how well they can withstand fatigue under various levels of stress and temperature. The fatigue phenomenon which occurs is defined as the process of progressive localized permanent structural change occurring in a material subjected to conditions which produce fluctuating stresses at some point or points in the material and may result in cracking or complete failure of the material being stressed.

The purpose of this procedure is to describe the methodology for the planning of tests to produce fatigue life data and the statistical techniques for analyzing the results of such tests. The requirement for this procedure is set forth in Data Item R101, NERVA Reliability Program Plan.

2.0 APPLICABLE DOCUMENTS

2.1 Data Item R101, Reliability Program Plan

2.2 Data Item R-106, Reliability Test and Evaluation Plan

2.3 NRP-401, Applicable Strength Theory for Selected Failure Modes

2.4 NRP-406, Reliability Calculations for Cases of Combined Stress and Fatigue Loading

2.5 NRP-600, Statistical Distributions, Their Applications and Tables

2.6 NRP-601, Error in Assumption of Normality

3.0 POLICY

3.1 The fatigue life of materials is an integral part of the design for reliability process during both the pre-design analytical activities and the post-design test and evaluation activities.

3.2 Proper interpretation and application of the methods described herein are essential for all engineers influencing the design and analysis of materials tests and utilizing the results of such tests.

4.0 DEFINITIONS

4.1 FATIGUE

Fatigue is the fracture of a structure due to initiation and progression of a micro-crack generated by repetitive (cyclic) variations in stresses and strains.

4.2 ENDURANCE LIMIT

That critical cyclic stress level for a particular structure below which repetitive cyclic loads do not cause observable fatigue damage.

4.3 MEAN LOG CYCLE LIFE

The average log cycle life for all possible specimens of the material tested. f(S) is the mean log cycle life at stress level S. fy(S) denotes an estimate of f(S), based on experimental data.

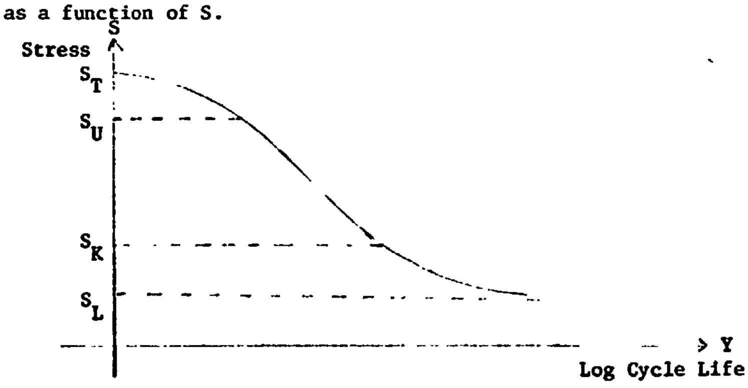
4.4 P%/γ% Lower Tolerance Boundary for Log Cycle Life

The P%/γ% lower tolerance boundary for log cycle life is a function, L(S), of stress such that with confidence γ% (e.g., $\gamma = 95$), P% (e.g., $P = 99$) of the population of specimens has a log cycle life, at stress level S, greater than L(S).

5.0 PROCEDURE

5.1 INTRODUCTION

In fatigue testing, a specimen is put on a machine at a fixed temperature and subjected to stress cycles until it breaks or fails in some defined way. Let S be the stress applied to the specimen during each stress cycle, N be the number of stress cycles to specimen failure, and Y be the logarithm of N. It is known that there is a strong relationship between Y and S. In fact, the sketch below is a typical graph of Y as a function of S.



Stress Versus Log Cycle Life Relationship

Figure 1

5.1, Introduction (cont.)

In practice, of course, due to variability of specimens and experimental conditions, the data points (S, Y) do not lie on a smooth curve. Instead, they lie on a band having the same shape as the curve in the preceding graph. Actually, the curve in Figure 1 is a sketch of the mean or average log cycle life of specimens as a function of S. This function will be called f(S). f(S) is an unknown function which is to be estimated from fatigue data.

The four points marked on the vertical axis in Figure 1 have special meanings. S_L is the stress below which the fatigue life of specimens is infinite, for practical purposes. S_L is called the fatigue or endurance limit of the material. S_K is a stress level near which the mechanism of fatigue failure changes, leading to a different slope in the Y versus S relationship. S_K is called the "knee" stress of the graph of the true mean log cycle life, f(S). S_U is a point near which failure begins to occur as a result of lack of strength rather than lack of endurance; and finally, S_T is the ultimate strength of the material. In fatigue testing, data are often collected for stresses between S_L and S_U.

Since it is known that there is a strong relationship between S and Y, it is assumed that

$$Y = f(S) + e \tag{1}$$

where f(S) is the true mean log cycle life, at stress S for all specimens, and e is an error term which reflects the scatter in fatigue data. (1) is called a regression model.

5.2 PLANNING A FATIGUE EXPERIMENT

5.2.1 Basic Quantities to be Determined

The statistical analysis of the data from a fatigue experiment will involve obtaining, for each temperature and stress of interest: (a) an estimate, $f_Y(S)$, of f(S); (b) an estimate, $\hat{\sigma}$ of the variance of $f_Y(S)$; and (c) a number k(S) such that if

$$L(S) = f_Y(S) - k(S)\,\hat{\sigma},$$

then L(S) is a P%/γ% lower tolerance boundary for log cycle life of specimens as a function of stress.

If the experiment is planned so that the quantities in (a), (b) and (c) can be determined from the experimental data, then all other statistical quantities describing the fatigue properties of specimens of the parent material can be obtained from data of the experiment.

5.2, Planning a Fatigue Experiment (cont.)

5.2.2 Determination of Test Stress Levels

Pilot studies to determine stress levels for fatigue testing at a given temperature are required if such test levels have not been established by prior information. Some knowledge of the shape of the true mean log cycle life for each temperature to be tested is desirable so that the experiment can be planned to produce data which yields a good estimate of $f_Y(S)$. Having rough knowledge of the quantities S_L, S_U, S_K, and S_T would be sufficient to plan the experiment. S_T can be roughly determined by cycling a few specimens at the minimum stress necessary to fail them in one stress cycle or less; then S_T would be the average breaking strength of the specimens tested in this way.

At this point the investigator should choose the maximum number, N_{max}, of cycles he is willing to let specimens run on his testing machines. A specimen surviving N_{max} stress cycles is called a runout. N_{max} should be as large as possible and certainly larger than cycle lives in the region of interest. For example, if he wished to determine whether a material will survive 10^7 stress cycles at a low stress level, then N_{max} should be greater than 10^7, so that some specimens will fail at greater than 10^7 cycles during the experiment. From the point of view of statistical analysis it is more valuable to know the cycle life until failure of a specimen than to know that a specimen survived a given number of cycles.

Having selected N_{max}, a rough estimate of S_L should be obtained. One method for doing this would be to test the first specimen at 1/2 S_T; if a failure occurs, test the second specimen at 1/4 S_T; and if the first specimen survives N_{max} stress cycles, test the second specimen at 3/4 S_T. Continue testing in this way until, say, 10 specimens have been tested. S_L can then be roughly estimated as the greatest stress at or below which all specimens were runouts. The accuracy of this estimate would be around $\pm S_T/2^{10}$. The investigator may be able to use his professional knowledge of the material being tested to shorten this procedure for estimating S_L and S_T, unless the investigator is interested in a narrower range, in which case he should only test at stresses in the range of interest.

The investigator should test more specimens at other stress levels, if necessary, to obtain fairly even spacing between the stress levels at which specimens have been tested. S_K can then be roughly estimated as that stress, in the lower part of the stress range, where the cycle life of specimens tested begins to increase sharply, as in Figure 1, as stress decreases. S_U can be similarly estimated as that stress, in the upper part of the stress range, where the cycle life of specimens tested begins to decrease sharply, as in Figure 1, as stress increases.

When selecting stress levels for testing, the investigator should keep in mind that one of the main goals of a fatigue experiment is to estimate the true mean log cycle life, $f(S)$, of specimens from the parent material. Thus, if S_K and S_U can be determined roughly, stress levels should be chosen so that the main experiment will produce enough data in each of the stress ranges S_L to S_K, S_K to S_U, and S_U to S_T, to estimate $f(S)$. The stress level should be equally spaced within each of the above ranges. At least three stress levels should be tested within each stress range above, if that range is tested.

5.2.2, Determination of Test Stress Levels (cont.)

It may well be that S_K and S_U do not exist, in the sense that it is not apparent from the data in this pilot study what the estimates of S_K and S_U are. This would happen if the change in cycle life, as stress decreases, were very gradual throughout the range from S_L to S_T. In this case, the testing in the main experiment should be done at eight or more stress levels evenly spaced between S_L and S_T, to ensure that there is data at enough stress levels to properly estimate f(S) throughout the range of stress levels from S_L to S_T.

5.2.2.1 Test Requirements at Stress Levels

Within each of the stress ranges, equal numbers of specimens should be tested at each stress level. At least five specimens should be tested at each stress level. More than five specimens should be tested at each selected stress level below S_K, if possible, because there is commonly more scatter in fatigue data for stresses below S_K. The purpose of testing at least five specimens per stress level is to allow the investigator to determine the relationship between the variance of log cycle life of the specimens and the stress applied to the specimens.

Often estimates of S_L and S_T for the material being tested are in the literature. The investigator should read this literature (if it is available) before doing any testing. In some cases, the investigator may not be interested in the fatigue properties of the material for stress levels outside of a certain range. In this case, he should follow the above procedure for allocating test specimens to stress levels, except that he should not test specimens at stress levels outside the range of interest.

5.2.3 <u>Temperature Selection and Selection of Stress Levels within Temperatures</u>

If it is believed that temperature will play an important role in the fatigue life of the materials, then the fatigue tests should be performed at each of three more equally spaced (if possible) temperatures within the temperature range of interest. The pilot study should be done at a medium temperature, or at the temperature, if one exists, of greatest interest to the investigator. Stress levels for the main experiment should be chosen as in Paragraph 5.2.2 for that temperature. The same stress levels should be tested at the other temperatures, to ensure that the fatigue data for different temperatures can be compared.

If only one temperature is of interest to the investigator, he should do all of his fatigue testing at that temperature.

5.2, Planning a Fatigue Experiment (cont.)

5.2.4 Planning the Experiment to Take Lot Effects into Account

Specimens are fabricated from different lots (heats, forgings, etc.) of parent metal. It is possible that different lots of specimens tend to have different average fatigue lives. Thus, different lots of specimens should be tested, so that lot-to-lot variability can be estimated and taken into account. The same number of specimens from each lot should be tested at each stress level-temperature combination in the experiment. If the investigator cannot (due to lot size limitations) test the same number of specimens from each lot at each stress level temperature combination, he should come as close as he can to meeting this requirement (cf. equation 29). In line with this, the most serious error would be to test at high stress levels with one lot of specimens and at low stress levels with another lot; for in such an experiment, the lot effect on fatigue life could not be separated from the stress effect on fatigue life. The investigator should test at least two specimens from each lot at each stress and temperature level tested.

5.2.5 Planning the Fatigue Experiment to Eliminate Bias in the Fatigue Data Due to Other Effects

Conceivably, factors such as the testing machines used in the experiments, the machine operators, the time of day, position of specimens within a lot, and so on can influence the observed fatigue life of specimens. One could call these effects bias factors. To minimize the influence in the experimental results due to bias factors, the investigator should randomly allocate lot-stress-temperature combinations to bias factor combinations.

5.2.6 Notching Effect

If little is known about the effect of notching on the cycle life of specimens of the material being tested, the investigator should test notched specimens at the stress and temperature levels at which he tests unnotched specimens. Fewer notched specimens would be tested than unnotched specimens, if professional knowledge were available to help the investigator estimate the mean log cycle life, f(S), for notched data, using the estimate of f(S) obtained for unnotched data.

5.2.7 Frequency Effect

It is conceivable that the number of stress cycles applied to the specimen per unit of time (i.e., the frequency) could influence the cycle-life of a specimen. The conditions under which it is known that frequency influences the cycle life of specimens are:

(a) At stress levels in the neighborhood of the ultimate strength of the material, cycling at high frequencies could cause a buildup of temperature in a structure which, if not dissipated, could lead to shortened cycle life of the structure.

5.2.7, Frequency Effect (cont.)

(b) At stress levels near the endurance limit of the material, cycling at high frequencies could lengthen the cycle life of a structure, since, at high frequencies, the structure may not have time to react to the imposed stress condition during each stress cycle.

It is known that frequency has no effect on cycle life of specimens tested at stress levels well below their ultimate strength and at frequencies between 200 and 5,000 cycles per minute. (cf ref. 3, 4, 5)

It is expected that the frequency effect on cycle life will be insignificant for frequencies up to 30,000 cycles per minute. Thus, if it is anticipated that the material will undergo conditions (a) or (b) for frequencies outside the range from 200 to 30,000 cpm., then the fatigue tests should take place under conditions and frequencies as close as possible to those to be encountered in service.

5.2.8 Other Effects Which can Significantly Influence the Fatigue Life of a Material

The investigator should be cognizant of the fact that surface effects can have an influence on the fatigue life of the material. Some surface effects are:

a. Grinding
b. Machining
c. Rolling
d. Polishing
e. Coating
f. Plating
g. Carburizing and/or nitriding
h. Pickling
i. Stress relieving
j. Roughness.

Other factors which can influence the fatigue life of the material are:

k. Conditions which cause creep
l. Conditions which cause corrosion
m. Size effects
n. Configuration effect
o. Loading effect.

5.2.8, Other Effects Which can Significantly Influence the Fatigue Life of a Material (cont.)

If the investigator anticipates that any of the above effects will influence the fatigue life of the material he is testing, and if there is little or no professional knowledge which enables the investigator to predict the influence of these effects on fatigue life, then the investigator should test specimens which are as close as possible to being like the parts to be used in service, under conditions as close as possible to those to be encountered in service.

5.3 STATISTICAL ANALYSIS

Appendix A contains an example of typical calculations which are given in this section.

5.3.1 The Probability Distribution of Log Cycle Life at a Given Stress, S, for a Fixed Temperature

It will be assumed that the log cycle life of a specimen at given stress S is normally distributed with unknown mean, f(S), and unknown variance σ^2, where σ^2 may depend on S. This assumption is common in fatigue literature. Some authors have used the Weibull distribution for cycle life. However, it has been found (for example see Reference [2]), that the assumption of a normal distribution for log cycle life is reasonable; moreover, there are fewer statistical techniques available for analyzing fatigue data on the basis of the assumption of a Weibull distribution for cycle life. Further, the only way to tell which distribution should be used would be to test many (25 or more) specimens at a given stress level and then plot cycle life versus percent survival on log normal paper and on Weibull paper and see which plot is more linear. Rather than having too much concern about the distributional form of cycle life, it is generally more worthwhile to test fewer specimens at each stress level, and instead to test at more temperatures and stress levels, to get a better idea of the shape of the stress versus log cycle life relationship at more temperatures. It is anticipated that the estimates of f(S), given in this procedure, will not be significantly affected by non-normality in the distribution of log cycle life. However, non-normality in the distribution of log cycle life could affect the validity of estimates of lower tolerance limits for log cycle life. The estimates of lower tolerance limits could be too high or too low depending on the true distribution log cycle life (cf. NRP-601, Paragraphs 4.7.3 and 4.7.4 for a general discussion of the effects of non-normality on lower tolerance limits).

5.3.2 Plotting the Data

Using separate graphs for each temperature tested, plot the points (Y, S), where Y is the log cycle life of a specimen tested at stress S. Plot Y on the horizontal axis and S on the vertical axis. Identify the points (Y, S) (e.g., by color) by the lot from which the corresponding specimens were chosen.

5.3, Statistical Analysis (cont.)

5.3.3 Estimating the Mean Log Cycle Life, f(S), and a Lower Tolerance Boundary, L(S), for Log Cycle Life at a Fixed Temperature

The methods used to estimate f(S) and L(S) depends upon the nature of the data in the scatter diagram plotted as in Paragraph 5.3.2. In Paragraphs 5.3.3.1 and 5.3.3.2, methods are presented to estimate f(S) and L(S) together with the criteria to be satisfied for use of these methods. These criteria are based on the quantities defined beginning with (2) below, where S_1 is the lowest stress at which no runouts occurred, and $S_1 < S_2 < \ldots < S_p$. Using these quantities, the investigator should choose one of three methods for estimating f(S) and L(S):

a. Linear regression method, where the mean log cycle life is assumed to be a linear function stress for stress levels both above the "knee", S_K, and below S_K (S_K will be defined). This method is presented in Paragraphs 5.3.3.1 through 5.3.3.1.3.

b. A more general linear regression method, where it is assumed that the mean log cycle is linearly related to a function of stress. This method is presented in Paragraph 5.3.3.1.4.

c. A method which consists of fitting (by eyeball) a curve to the experimental mean log cycle lives to estimate f(S), and computing the corresponding lower tolerance boundary L(S). This method is presented in Paragraph 5.3.3.2.

Method a. should be used when the criteria in Paragraph 5.3.3.1.2 are satisfied. If these criteria are not satisfied, method b. should be used. If there is no function of stress which provides a good fit to the experimental mean log cycle lives, method c. should be used. Appendix A will provide an example of the results which can be obtained using these methods.

Let there be N lots, and let n_{ij} = the number of specimens in the j^{th} lot tested at the i^{th} stress level, (2)

$$n_i = \sum_{j=1}^{m} n_{ij}, \tag{3}$$

$$N = \sum_{i=1}^{p} n_i, \tag{4}$$

$$\bar{Y}_i = \sum_{j=1}^{m} \sum_{k=1}^{n_{ij}} Y_{ijk}/n_i, \tag{5}$$

where Y_{ijk} is the log cycle life of the k^{th} specimen, from the j^{th} lot, tested at the i^{th} stress level, S_i, and $i=1,\ldots,p$, $j=1,\ldots,m$, and $k=1,\ldots,n_{ij}$.

5.3.3. Estimating the Mean Log Cycle Life, f(S), and a Lower Tolerance Boundary, L(S), for Log Cycle Life at a Fixed Temperature (cont.)

Let

$$f_{ij} = n_{ij} - 1, \tag{6}$$

$$f_i = \sum_{j=1}^{m} f_{ij}, \tag{7}$$

$$\overline{Y}_{ij} = \sum_{k=1}^{n_{ij}} Y_{ijk}/n_{ij}, \tag{8}$$

$$s_{ij}^2 = \frac{\sum_{k=1}^{n} \left(Y_{ijk} - \overline{Y}_{ij}\right)^2}{f_{ij}}, \tag{9}$$

$$s_i^2 = \frac{\sum_{j=1}^{m} f_{ij}\, s_{ij}^2}{f_i}. \tag{10}$$

Then s_i^2 is then an estimate of the variance in log cycle life due to within-lot variability, at stress S_i. Let

$$s_{L_i}^2 = \frac{\sum_{j=1}^{m} n_{ij} \left(\overline{Y}_{ij} - \overline{Y}_i\right)^2}{(m-1)}. \tag{11}$$

If

$$\hat{\sigma}_i^2 = [(n_i' - 1)\, s_i^2 + s_{L_i}^2]/n_i',$$

where

$$n_i' = (n_i^2 - \sum_{j=1}^{m} n_{ij}^2)/[n_i\,(m-1)], \tag{12}$$

then $\hat{\sigma}_i^2$ is an estimate of the variance of log cycle life at stress level S_i taking both within lot and lot-to-lot variability into account. (Note: if there is only one lot available, set

$$\hat{\sigma}_i^2 = s_i^2 + \sigma_\beta^2,$$

where σ_β^2 is the investigator's estimate of the lot-to-lot variance of log cycle life of specimens.)

5.3.3, Estimating the Mean Log Cycle Life, f(S), and a Lower Tolerance Boundary, L(S), for Log Cycle Life at a Fixed Temperature (cont.)

At each stress level, S, below S_1, but at which at least half of the specimens were non-runouts, let $\overline{Y}_S$ be the sample median log cycle life, computed as follows:

Let $Y_1, Y_2, \ldots, Y_n$ be the log cycle lives, until failure or runout, of the specimens tested at stress level S. Then let $z_{(i)}$, i=1, 2, ..., n, be the i^{th} largest of the Y_i's. Then

$$\overline{Y}_S = z_{(\frac{n+1}{2})} \text{ if n is odd}$$

$$= \frac{z_{(\frac{n}{2})} + z_{(\frac{n+1}{2})}}{2} \text{ if n is even} \qquad (13)$$

$\overline{Y}_S$ is a fairly good estimate of the mean log cycle life at stress level S.

5.3.3.1 The Linear Regression Method for Estimating the Mean Log Cycle Life of Specimens, f(S), and a Lower Tolerance Boundary, L(S), for Log Cycle Life of Specimens

5.3.3.1.1 Plot the points $(\overline{Y}_i, S_i)$ and the points $(\overline{Y}_S, S)$ on a graph with the $\overline{Y}$'s as the abcissas and S's as the ordinates. Draw a smooth curve $f_Y(S)$, fitting these points as closely as possible. Also, on the same graph draw a smooth curve, $f_L(S)$, fitting the points $(Y_i - \hat{\sigma}_i, S_i)$ as closely as possible, and continuing the curve to the lowest stress level tested, $f_Y(S)$ is then a good estimate of mean log cycle life, f(S), at stress S. $f_L(S)$ is an estimate of the 84% survival point, $L_{84}(S)$, at stress S. $L_{84}(S)$ has the property that all specimens have an 84% chance of having a log cycle life greater than $L_{84}(S)$.

5.3.3.1.2 If a., b., and c. of the following criteria are met, then linear regression should be used to estimate f(S) and L(S). If any of these criteria fail, then f(S) should be estimated by $f_Y(S)$ as above, and L(S) should be estimated as in paragraph 5.3.3.2, or $f_Y(S)$ and L(S) should be estimated as in Paragraph 5.3.3.1.4.

a. $f_Y(S)$ should have a sharp bend in the lower stress range, in the vicinity of a stress level (called the knee) S_K.

b. $f_Y(S)$ should be nearly linear above S_K and below S_K. (This means, in particular, that there is no sharp bend in $f_Y(S)$ in the higher part of the stress range tested.)

c. The "bandwidth", $f_Y(S) - f_L(S)$, should increase sharply in the vicinity of knee, S_K, as stress decreases. Remark: S_K should be chosen so that it is between two test stress levels, thus dividing the fatigue data into two sets; also there should be few runouts at stresses above S_K.

5.3.3. Estimating the Mean Log Cycle Life, f(S), and a Lower Tolerance Boundary, L(S), for Log Cycle Life at a Fixed Temperature (cont.)

5.3.3.1.3 By criterion b. of Paragraph 5.3.3.1.2 it is reasonable to assume that

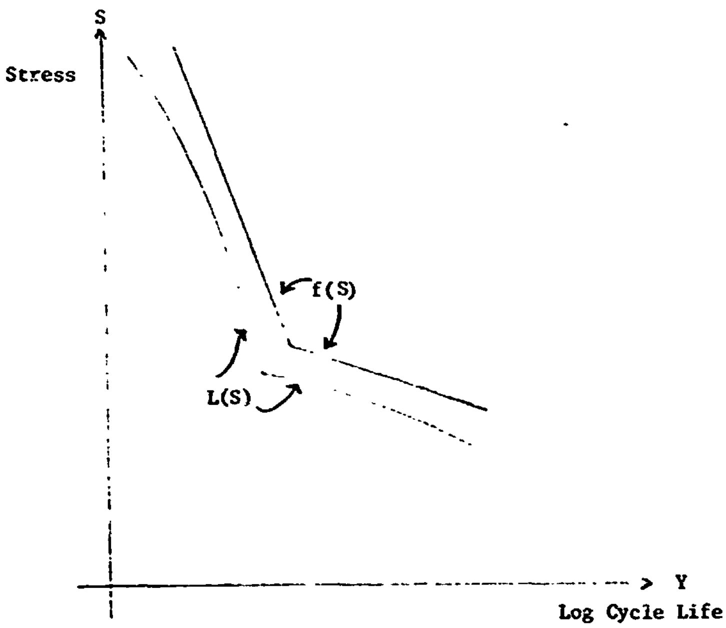
$$f(S) = a_1 + b_1 S \text{ if } S \geq S_K, \text{ or} \tag{14}$$

$$f(S) = a_2 + b_2 S \text{ if } S_L < S < S_K. \tag{15}$$

Where S_K is estimated as in Paragraph 5.3.3.1.2, and S_L, the endurance limit of specimens of the material is estimated as that stress level, at and below which, all specimens in the fatigue exoeriment were runouts.

Using assumptions (14) and (15), methods will be presented for obtaining estimates, $f_Y(S)$ and L(S), of the mean log cycle life and lower tolerance boundary for log life of specimens of the parent material.

These estimates can then be graphed, and the resulting graph will look like Figure 2:



Estimated Mean by Cycle Life versus Stress, $f_Y(S)$ and Lower Tolerance Boundary, L(S)

Figure 2

5.3.3, Estimating the Mean Log Cycle Life, $f(S)$, and a Lower Tolerance Boundary, $L(S)$, for Log Cycle List at a Fixed Temperature (cont.)

To obtain estimates $\hat{a}_1$ and $\hat{b}_1$ of a_1 and b_1, in Equation (14) perform the calculations given below:

$$f_Y(S) = \hat{a}_1 + \hat{b}_1 S$$

will then be the "least squares" regression line for mean log cycle life versus stress; $f_Y(S)$ will also be an estimate of $f(S)$ for all $S \geq S_K$. Let

$$S_1, S_2, \ldots, S_v$$

be the stress levels tested in the stress range above S_K. Let Y_{ijk} be the cycle life of the k^{th} specimen from the j^{th} lot, tested at the i^{th} stress level. Assume i varies from 1 to v, j varies from 1 to m, (m would be one if only one lot were tested) k varies from 1 to n_{ij}, where n_{ij} is the number of specimens tested in the j^{th} lot at the i^{th} stress level.

Let

$$N = \sum_{i=1}^{v} \sum_{j=1}^{m} n_{ij} \tag{16}$$

$$n_j = \sum_{i=1}^{v} n_{ij} \tag{17}$$

$$\bar{Y}_j = \sum_{i=1}^{v} \sum_{k=1}^{n_{ij}} Y_{ijk}/n_j \tag{18}$$

$$\bar{S}_j = \sum_{i=1}^{v} n_{ij} S_i/n_j \tag{19}$$

$$\hat{\beta}_j = \frac{\sum_{i=1}^{v} \sum_{k=1}^{n_{ij}} \left(Y_{ijk} - \bar{Y}_j\right) \left(S_i - \bar{S}_j\right)}{\sum_{i=1}^{v} n_{ij} \left(S_i - \bar{S}_j\right)^2} \tag{20}$$

$$\hat{\alpha}_j = \bar{Y}_j - \hat{\beta}_j \bar{S}_j \tag{21}$$

5.3.3. Estimating the Mean Log Cycle Life, f(S), and a Lower Tolerance Boundary, L(S), for Log Cycle List at a Fixed Temperature (cont.)

Then $\hat{\alpha}_j + \hat{\beta}_j S$ is an estimate of the mean log cycle life at stress $S \geq S_K$, for specimens from the j^{th} lot; moreover,

$$\hat{Y}_j = \hat{\alpha}_j + \hat{\beta}_j S$$

is the least squares regression line for log cycle life versus stress for specimens from the j^{th} lot. The within-lot variance of log cycle life, for the j^{th} lot, should be estimated by

$$\hat{\sigma}_j^2 = \frac{\sum_{i=1}^{v} \sum_{k=1}^{n_{ij}} \left(Y_{ijk} - \overline{Y}_j\right)^2 - \hat{\beta}_j \sum_{i=1}^{v} \sum_{k=1}^{n_{ij}} \left(Y_{ijk} - \overline{Y}_j\right)\left(S_i - \overline{S}_j\right)}{n_j - 2} \tag{22}$$

To estimate the mean log cycle life for all lots, let

$$\hat{a}_1 = \frac{\sum_{j=1}^{m} n_j \hat{\alpha}_j}{N} \tag{23}$$

$$\hat{b}_1 = \frac{\sum_{j=1}^{m} n_j \hat{\beta}_j}{N} \tag{24}$$

Then $f_Y(S) = \hat{a}_1 + \hat{b}_1 S$ for $S \geq S_K$ is an estimate of the mean log cycle life, f(S) for specimens cycled at stress S.

The component of variance in log cycle life due to within-lot variation should be estimated by

$$\hat{\sigma}_w = \frac{\sum_{j=1}^{m} \left(n_j - 2\right) \hat{\sigma}_j^2}{N - 2m} \tag{26}$$

Let

$$n_o = \frac{N^2 - \sum_{j=1}^{m} n_j^2}{Nm - N} \tag{27}$$

$$\overline{Y} = \frac{\sum_{j=1}^{m} n_j \overline{Y}_j}{N}, \tag{28}$$

5.3.3. Estimating the Mean Log Cycle Life, f(S), and a Lower Tolerance Boundary, L(S), for Log Cycle List at a Fixed Temperature (cont.)

and

$$\hat{\sigma}_L^2 = \sum_{j=1}^{m} \frac{n_j \left(\overline{Y}_j - \overline{Y}\right)^2}{m - 1}; \qquad (29)$$

then

$$\hat{\sigma}^2 = \frac{\hat{\sigma}_L^2}{n_o} + \frac{n_o - 1}{n_o} \hat{\sigma}_w^2 \qquad (30)$$

is an estimate of the variance in log cycle life of specimens which takes both within-lot and lot-to-lot variation into account.

Notice that in equation (29) the within-lot average lot cycle lives, $\overline{Y}_j$, are compared with the overall average log cycle life, $\overline{Y}$. This equation will be wrong unless nearly equal numbers of specimens are tested at each stress level from each lot. Now, the calculations required to produce the lower tolerance boundary L(S) will be presented. Assume that the investigator desires that at least P% (e.g., P = 99) of all possible specimens of the material have log cycle life, at stress S or less, greater than a certain quantity with confidence γ% (e.g., γ = 95). Then the quantity, L(S) will have this property. Set

$$A_j(S) = \frac{1}{n_j} + \frac{(\overline{S}_j - S)^2}{\sum_{i=1}^{v} n_{ij} (S_i - \overline{S}_j)^2}, \qquad (31)$$

n = greatest integer not greater than

$$\frac{N^2 \hat{\sigma}^2}{\sum_{j=1}^{m} \left[\left(\frac{\hat{\sigma}_L^2}{n_o} - \frac{1}{n_o} \hat{\sigma}_w^2\right) + \hat{\sigma}_w^2 A_j(S)\right] n_j^2}, \qquad (32)$$

f = greatest integer not greater than

$$\frac{\left(\hat{\sigma}^2\right)^2}{\frac{\hat{\sigma}_L^2}{n_o^2 (m-1)} + \frac{\left(\frac{n_o - 1}{n_o}\right)^2 \left(\hat{\sigma}_w^2\right)^2}{N - 2m}}. \qquad (33)$$

Set

$$L(S) = \hat{a}_1 + \hat{b}_1 S - k(S)\hat{\sigma}, \text{ for } S \geq S_K \qquad (34)$$

5.3.3. Estimating the Mean Log Cycle Life, f(S), and a Lower Tolerance Boundary, L(S), for Log Cycle List at a Fixed Temperature (cont.)

where k(S) is the one-sided P%/γ% tolerance factor corresponding to f and n, which can be obtained from Reference [1]. If $m = 1$, i.e., only one lot was tested, then the following alternate method for calculating L(S) should be used. Set

$$\sigma_B^2 = \text{upper bound on lot-to-lot variation, to be} \qquad (36)$$

determined by the investigator using his knowledge and experience, (24)

$$f = N-2m \qquad (37)$$

and (25) n = greatest integer not greater than

$$\frac{\left(\hat{\sigma}_w^2 + \hat{\sigma}_B^2\right) N2}{\sum_{j=1}^{m} \sigma_B^2 + \sigma_w^2 A_j(S) \quad n_j^{\ 2}} \qquad (38)$$

(note, in this case $m = 1$). Then

$$L(S) = a_1 + b_1 S - k(S) \ (\sigma_B^2 + \hat{\sigma}_w^2)^{1/2}, \ S > S_K \qquad (39)$$

is a P%/γ% lower tolerance boundary for the log cycle life of specimen of the parent material.

It may happen due to the number, m, of lots being small, that f in (33) is so small that L(S) is unreasonably low (e.g., $L(S) < o$). In this case, the investigator may use (38) to estimate (37) and (39) can then be used to estimate L(S). Also there may be a few runouts at stresses only a little above S_K. If there are only a few runouts (as there should be, by choice of S_K), then it is reasonable to treat these few runout specimens as if they had failed at N_{max} stress cycles and analyze the resulting data as in (2) through (39). The resulting mean log cycle life estimate, $\hat{a}_1 + \hat{b}_1 S$, can be either slightly too high or slightly too low; the same is true of L(S).

5.3.3, Estimating the Mean Log Cycle Life, f(S), and a Lower Tolerance Boundary, L(S), for Log Cycle List at a Fixed Temperature (cont.)

To estimate $a_2 + b_2$ S and L(S) for $S < S_K$, the same procedure should be followed as in estimating (14) if only a few specimens tested at stresses below S_K are runouts. Also, if only a few runouts occurred at all stress levels tested, above a certain stress, S_{min}, then (15) and (34) or (39) for $S < S_K$, can be estimated using the same procedure as in estimating (15), if at least three stress levels between S_{min} and S_K, were tested. The estimates of (14) and (34) or (39) for $S < S_K$ can then be extrapolated downward to obtain conservative estimates of the mean log cycle life and L(S) for $S < S_K$. However, if many of the specimens tested at stress levels below S_K were runouts, then the methods given in Paragraph 5.3.3.3 should be used for estimating f(S) and L(S) for $S < S_K$.

Finally, the procedure in this section for estimating f(S) and L(S) should be modified if there is a strong relationship between the S_i of Paragraph 5.3.3, equation (10) and stress S_i. This relationship can be determined by plotting s_i versus S_i and fitting a smooth curve $s = rg(S)$ to this plot where r is a constant. Then the log cycle life data can be weighed so that the sample variances of the weighted data are fairly constant.

$g(S_i)$ should be a very simple function, which increases as S_i decreases. Some candidates for $g(S_i)$ are:

$$g(S_i) = c - S_i, \tag{40}$$

$$g(S_i) = -\log S_i + c, \tag{41}$$

$$g(S_i) = -\sqrt{S_i + c}, \tag{42}$$

$$g(S_i) = \frac{1}{S_i}, \tag{43}$$

and so on. If $g(S_i)$ is one of the above functions, or some other function which the investigator may want to use, then r is chosen to be the positive constant such that the curve $s = rg(S)$ fits the points (s_i, S_i) as closely as possible. Set

$$w_i = \frac{v\left(\frac{1}{g(S_i)}\right)^2}{\sum_{k=1}^{v}\left(\frac{1}{g(S_i)}\right)^2}\ ; \tag{44}$$

the w_i's are the weighting coefficients. Replace (18) by

$$\bar{Y}_j = \left(\sum_{i=1}^{v}\sum_{k=1}^{n_{ij}} w_i Y_{ijk}\right)\Big/\left(\sum_{i=1}^{v} w_i n_{ij}\right) \tag{45}$$

5.3.3, Estimating the Mean Log Cycle Life, f(S), and a Lower Tolerance Boundary, L(S), for Log Cycle List at a Fixed Temperature (cont.)

Replace (19) by

$$\bar{S}_j = \left(\sum_{i=1}^{v} n_{ij} w_i S_i\right) / \left(\sum_{i=1}^{v} w_i n_{ij}\right) . \tag{46}$$

Replace (20) by

$$\hat{\beta}_j = \frac{\sum_{i=1}^{v} \sum_{k=1}^{n_{ij}} w_i (Y_{ijk} - \bar{Y}_j)(S_i - \bar{S}_j)}{\sum_{i=1}^{v} w_i n_{ij} \left(S_i - \bar{S}_j\right)^2} \tag{47}$$

$\hat{\alpha}_j$ should be computed as in (21), using (45), (46), and (47) (48)

Replace (22) by

$$\hat{\sigma}_j^2 = \sum_{i=1}^{v} \frac{\sum_{k=1}^{n_{ij}} w_i (Y_{ijk} - \hat{\alpha}_j - \hat{\beta}_j S_i)^2}{\left(\sum_{i=1}^{k} n_{ij}\right) - 2} \tag{49}$$

using (47) and (48).

Calculate (23) through (29) as before, using the substitutions (47), (48) and (49). Set

$$n_o' = \sum_{j=1}^{m} \sum_{i=1}^{v} \frac{n_{ij}}{w_i} \left(\frac{1}{n_j} - \frac{1}{N}\right)/(m - 1) \tag{50}$$

Replace (30) by

$$\hat{\sigma}^2 = \frac{\hat{\sigma}_L^2}{n_o} + \left(1 - \frac{n_o'}{n_o}\right) \hat{\sigma}_w^2 \tag{51}$$

Replace (31) by

$$A_j(S) = \frac{1}{\sum_{i=1}^{v} w_i n_{ij}} + \frac{(S - \bar{S}_j)^2}{\sum_{i=1}^{v} w_i n_{ij} (S_i - \bar{S}_j)^2} \tag{52}$$

where the $\bar{S}_j$'s are as in (16).

5.3.3, Estimating the Mean Log Cycle Life, f(S), and a Lower Tolerance Boundary, L(S), for Log Cycle List at a Fixed Temperature (cont.)

Calculate (32) as before, using the "new" values of $\hat{\sigma}_w^2$ and $\hat{\sigma}_L^2$ in computing n. Replace (33) with

f = greatest integer not greater than

$$\frac{(\hat{\sigma}^2)^2}{\dfrac{(\hat{\sigma}_L^2)^2}{n_o^2\,(m-1)} + \left(1 - \dfrac{n'_o}{n_o}\right)^2 \dfrac{(\hat{\sigma}_w^2)^2}{N-2m}} \tag{53}$$

using the "new" values of $\hat{\sigma}^2$ and $\hat{\sigma}_w^2$ to calculate f.

Calculate L(S) exactly as in (34), using the new n, $\hat{\sigma}^2$, $\hat{a}_1$, and $\hat{b}_1$.

5.3.3.1.4 Estimation of f(S) and L(S) when the Mean Log Cycle Life is Linearly Related to a Function of Stress

It may be true that for all S in the range of stresses tested,

$$f(S) = a + b\, g(S), \tag{54}$$

where g is some function of S. A prime candidate for g is

$$g(S) = \log S. \tag{55}$$

If it is suspected that a relationship of the form in (54) holds, then the log cycle life versus g(S) data should be plotted, so that the investigator can check whether or not (54) holds. If the resulting plot is fairly linear, then a, b, and a lower tolerance boundary L (g(S)) for log cycle life as a function of g(S), can be estimated using linear regression as in Paragraph 5.3.3.1.3, with "g(S)" in place of "S", except that only one pair (a, b) of parameters is estimated using data for all stress levels tested.

5.3.3.2 The Estimation of f(S) and L(S) When the Linear Regression Method is not Appropriate

5.3.3.2.1 The $f_Y(S)$ of Paragraph 5.3.3.1.1 should be used to estimate the mean log cycle life, f(S), when the criteria for using linear regression are not met, and $S > S_1$. Let

$$s^2 = \sum_{i=1}^{p} \frac{f_i\, s_i^2}{\sum_{i=1}^{n} f_i} \tag{56}$$

5.3.3. Estimating the Mean Log Cycle Life, f(S), and a Lower Tolerance Boundary, L(S), for Log Cycle List at a Fixed Temperature (cont.)

where s_i^2 is calculated as in (10) and f_i is calculated as in (7). Let

$$\hat{\sigma}^2 = \frac{\hat{\sigma}_L^2}{n_o} + \frac{n_o - 1}{n_o} s^2 \tag{57}$$

where $\hat{\sigma}_L^2$ is calculated as in (29) and n_o is calculated as in (27).

Let f = the greatest integer not greater than

$$\frac{(\hat{\sigma}^2)^2}{\frac{(\hat{\sigma}_L^2)^2}{n_o^2\,(m-1)} + \frac{\left(\frac{n_o - 1}{n_o}\right)^2 (s^2)^2}{\sum_{i=1}^{p} f_i}} \tag{58}$$

Let n_i^* = the greatest integer not greater than

$$\frac{\hat{\sigma}^2}{\frac{1}{n_i}\left[s^2 + \frac{\sum_{j=1}^{m} n_{ij}^2}{n_i}\left(\frac{\hat{\sigma}_L^2 - s^2}{n_o}\right)\right]} \tag{59}$$

where n_i is calculated as in (3), and n_{ij} is defined as in (2).

If k is the P%/γ% one-sided tolerance factor corresponding to f and n_i^* in the table (see Reference [1]), then

$$L(S_i) = f_Y(S_i) - k\hat{\sigma} \tag{60}$$

is a P%/γ% lower tolerance point for the log cycle life of specimens at stress S_i. A smooth curve should then be drawn so that it fits the points (S_i, L(S)) as closely as possible. This curve is the lower tolerance boundary L(S). As in Paragraph 5.3.3.1.3, there may be a function, g(S) [cf. (40), (41), (42) and (43)], giving the relationship between the stress level, S, and the within-lot standard deviation of log cycle life, which is estimated at the i^{th} stress level by S_i^2. Let w_i be calculated as in (44). Let n_o be calculated from (50). Let

$$s_w^2 = \frac{\sum_{i=1}^{p} w_i f_i s_i^2}{\sum_{i=1}^{p} f_i} \tag{61}$$

5.3.3, Estimating the Mean Log Cycle Life, f(S), and a Lower Tolerance Boundary, L(S), for Log Cycle List at a Fixed Temperature (cont.)

Let

$$\hat{\sigma}_{w_i}^{2} = \frac{\hat{\sigma}_L^{2}}{n_o} + \left(\frac{1}{w_i} - \frac{n_o'}{n_o}\right) s_w^{2} \tag{62}$$

where n_o, n_o', and $\hat{\sigma}_L$ are calculated from (27), (50) and (29).

Let f_i^{*} = the greatest integer not greater than

$$\frac{\left(\hat{\sigma}_{w_i}^{2}\right)}{\dfrac{(\hat{\sigma}_L^{2})^{2}}{n_o^{2}\,(m-1)} + \dfrac{\left(\frac{1}{w_i} - \frac{n_o'}{n_o}\right)^{2} (s_w^{2})^{2}}{f}} \tag{63}$$

and

$$n_i^{**} = \frac{\hat{\sigma}_{w_i}^{2}}{\dfrac{1}{n_i}\left[\dfrac{s_w^{2}}{w_i} + \sum_{j=1}^{m} \dfrac{n_{ij}^{2}}{n_i n_o} \quad (\hat{\sigma}_L^{2} - s_w^{2}\, n_o')\right]}$$

If k is the one-sided P%/Y% tolerance factor corresponding to n_i^{**} and f_i^{*} (cf. Reference [1] where $n = n_i^{**}$ and $f_i^{*} = f$), then

$$L(S_i) = f_y(S_i) - k\,\hat{\sigma}_{w_i}^{2}$$

is the P%/Y% lower tolerance boundary for log cycle life at stress level S_i. To obtain the lower tolerance boundary, L(S), a smooth curve should be drawn, fitting the points (S_i, $L(S_i)$) as closely as possible.

5.3.3.3 Estimating f(S) and L(S) for Stress Levels where Many Specimens were Runouts

5.3.3.3.1 The investigator should use the lower part of the curve $f_Y(S)$ to estimate f(S) at stress levels at which more than a few and not more than half of the specimens were runouts. He should then use his best judgment to extrapolate the lower part of the curve $f_Y(S)$, for lower stress levels. It would be conservative to perform this extrapolation using a line, with slope equal to the slope of the curve, $f_Y(S)$, at the lowest stress level at which not more than one half of the specimens were runouts. This is true because f(S) becomes less and less steep for lower and lower stresses.

5.3.3, Estimating the Mean Log Cycle Life, f(S), and a Lower Tolerance Boundary, L(S), for Log Cycle List at a Fixed Temperature (cont.)

5.3.3.3.2 The L(S) in Paragraph 5.3.3.1.3 or 5.3.3.2.1 should be extrapolated downward in the same way as $f_Y(S)$, to estimate the lower tolerance boundary for log cycle life, at stress levels at which many runouts occurred. Linear extrapolation of L(S) would be conservative for the same reason that linear extrapolation of $f_Y(S)$ is conservative.

5.3.4 Estimating the Lower Tolerance Boundary, L(T), for the Log Cycle Life of Specimens, at a Given Stress Level, S, for a Temperature T

Plot the points (T, L(S)), where L(S) is the lower tolerance boundary at stress level S and test temperature T, as calculated in Section 5.3.3. Draw a smooth curve, $L_t(T)$, which fits these points as closely as possible. Then $L_t(T)$ has the property that for the given stress, S, the proportion of specimens, with log cycle life greater than L(T) at temperature T, is at least P% with confidence γ%.

The same method should be used to estimate mean log cycle life $f_t(T)$, for specimens as a function of temperature, for a given stress level.

5.3.5 Estimating the Probability, R(No) for a Fixed Stress S and Temperature T, that a Specimen will Survive no Stress Cycles

Calculate an estimate, $\hat{f}_t(T)$, of $f_t(T)$ for the given stress as in Paragraph 5.3.4. Also, for the given stress, plot the bandwidth, $f_Y(S) - f_L(S)$ (cf. Paragraph 5.3.3.1.2 (c)) versus test temperature and draw a smooth curve, fitting this plot (plot temperature on the horizontal axis and bandwidth on the vertical axis). Then the distance, $\hat{\sigma}$, of this curve from the horizontal axis at temperature T will be an estimate of the standard deviation of log cycle life at stesss S and temperature T. Calculate

$$(1) \quad z = \frac{\text{Log No} - \hat{f}_t(T)}{\hat{\sigma}}$$

$$\text{Set} \quad (2) \quad R(\text{No}) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx$$

(R(No) can be found in the usual tables for the normal probability distribution.)

The following calculations lead to a lower γ% confidence boundary, R_L(No), for R(No) for a given temperature, T, and stress S.

$$\text{Set} \quad (3) \quad k = \frac{\text{Log No} - \hat{f}_t(T)}{\hat{\sigma}},$$

where $\hat{\sigma}$ is calculated appropriately as in Paragraph 5.3.3. Obtain n and f from the appropriate calculations of Paragraph 5.3.3 for the stress level and temperature nearest S and T. Let R_L(No) be the P- value corresponding to the one-sided tolerance factor k, and n, f, and γ%.

6.0 APPLICABILITY

6.1 This procedure should be used for planning and analyzing fatigue experiments.

6.2 Data Item R106 will define and identify the methods whereby fatigue data will be used in reliability assessment of the NERVA engine and its parts.

7.0 RESPONSIBILITIES

7.1 The proper use of the methods described herein are the responsibility of each cognizant engineer using these techniques. These will include:

7.1.1 Materials engineers in planning and analyzing fatigue experiments.

7.1.2 Reliability engineers in assessment of reliability.

7.1.3 Design engineers in using the results of fatigue experiments.

8.0 REFERENCES

[1] Owen, D. B., Factors for One-Sided Tolerance Limits and for Variables Sampling Plans, Sandia Corporation Monograph, March 1963

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[3] Grover, H. J., Gordon, S. A., and Jackson, L. R., Fatigue of Metals and Structures, Department of the Navy, Bureau of Aeronautics, 1954

[4] Forrest, P. G., Fatigue of Metals, Addison-Wesley Publishing Company, Inc., Palo Alto, 1962

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APPENDIX A

AN EXAMPLE, ILLUSTRATING METHODS OF
ANALYSIS OF FATIGUE DATA

Fatigue data was collected for aluminum specimens tested at 150°F (cf. Table 1). It was analyzed by using three different regression models to obtain estimates of the mean log cycle life, f(S), specimens as a function of stress, S, applied per cycle. Also, a lower 99/95 tolerance boundary, L(S), for log cycle life as a function of stress was obtained using each of the three models. L(S) has the property that with confidence 95% at least 99% of all specimens will have log cycle lives greater than L(S) at stress S and temperature 150°F. That is, with 95% confidence, the reliability is at least 99% that all specimens of the material will have log cycle lives greater than L(S). The three models are presented to illustrate the calculations used, and to show that there is quite a difference in the resulting lower tolerance boundaries for the three models. This difference indicates that care must be taken when choosing a model for statistical analysis of fatigue data.

Model 1 Fit Sample Mean Log Cycle Life vs Stress

Assumptions:

1. The mean log cycle life of a specimen is some unspecified function, f(S), of stress.

2. The log cycle life of a specimen is normally distributed.

That is, if Y is the log cycle life of a specimen, then

$$Y = f(S) + e$$

where e is a term which varies from specimen to specimen because of differences in specimens and experimental conditions. It is assumed that e is normally distributed with mean zero. Further, since, with two exceptions, the sample variances were homogeneous over the stress range tested (cf. Table 2, the "s_i^2" column) it was assumed that the variances of e were equal at all stress levels. Since the sample variance at 42.5 ksi was very small compared with the rest, and the sample variance at 27.5 ksi was large compared with the rest, the assumption of equal variances could lead to overly large values of the lower tolerance boundary for high stress level, and overly small values of the lower tolerance boundary for low stress levels.

The within-lot variance of log cycle life at the i^{th} stress level was estimated using

$$s_i^2 = \sum_{k=1}^{7} \frac{(Y_{ik} - \overline{Y}_i)^2}{6} ,$$

where Y_{ik} is the k^{th} observed log cycle life observed at the i^{th} stress level, and $\overline{Y}_i$ is the average log cycle life observed at the i^{th} stress level. These estimates were pooled to obtain an estimate, S^2, of the within-lot variance of log cycle life using

$$s^2 = \frac{\sum_{i=1}^{7} f_i s_i^2}{\sum_{i=1}^{7} f_i}$$

where f_i is the degrees of freedom for the sample variance, s_i^2 (f_i is n_i-1 where n_i is the sample size at the i^{th} stress level). To obtain an estimate of the mean log cycle life, f(S), the sample mean log cycle lives, $\overline{Y}_i$, were plotted vs. stress, $\hat{S}_i$. A curve, $\hat{f}_1(S)$, was drawn to fit the points ($\overline{Y}_i$, S_i) as closely as possible. $f_1(S)$ provides a reasonable estimate of f(S). The lower tolerance boundary was obtained by plotting the points $\hat{f}_1(S_i) - k_1\hat{\sigma}$ versus S_1 and fitting a curve to these points.

k_1 is the 99/90 tolerance factor, obtained from non-central t-distribution tables for n_i and $f = \sum_{i=1}^{7} f_i$ degrees of freedom. $\hat{\sigma}$ is the square root of the quantity s^2 plus σ_B^2. σ_B^2 is an estimate of the between lot variance of log cycle life; since only one lot was tested, σ_B^2 could not be estimated from the data. Thus, arbitrarily, for the sake of constructing this example, σ_B^2 was set equal to 0.05. In practice, if only one lot were tested, the investigator would have to use his best judgement to estimate σ_B^2; however, he should test more than one lot so that he can use the data to estimate σ_B^2.

<u>Model 2</u> Least Square Fit of Log Cycle Life to Two Regression Lines

This model is not appropriate for the data being examined, since there was no sharp bend in the curve $\hat{f}_1(S)$ as stress decreased. However, this method would be appropriate if there were a stress level, S_k, where

1. There were a sharp bend in the curve, $\hat{f}_1(S)$, near S_k.

2. $\hat{f}_1(S)$ has nearly constant slope above S_k and below S_k.

3. There were a sharp increase in the within lot variances, S_i^2, as the stress level went from above S_k to below S_k.

For the sake of an example illustrating this method, the computations were performed using Model 2.

Since there was no stress level S_k satisfying 1., 2., and 3., S_k in this example was somewhat arbitrarily chosen as 30 ksi. The data for stress levels above 30 ksi was put into one group and the rest of the data were put into another group. A regression line,

$$y_1 = a_1 + b_1 S \quad , \quad S > 30$$

was estimated from the log cycle life data in the above 30 ksi stress range, using the "least squares" method. Also, a regression line

$$y_2 = a_2 + b_2 S \quad , \quad S \leq 30$$

was estimated from the log cycle life data in the-at-or-below 30 ksi stress range, again the "least squares" method. The "least squares" estimate of b_1 was

$$\hat{b}_1 = \frac{\sum_{i=4}^{7} \sum_{k=1}^{n_i} (Y_{ik} - \bar{Y}_i)(S_i - \bar{S})}{\sum_{i=4}^{7} n_i (S_i - \bar{S})^2} = -0.0763$$

where

$$\bar{S} = \frac{\sum_{i=4}^{7} n_i S_i}{\sum_{i=4}^{7} n_i} = 37.5$$

The "least square" estimate of a_1 was

$$\hat{a}_1 = \bar{Y} - \hat{b}_1 \bar{S}$$

$$= 5.23 + 2.86 = 8.09$$

The estimate of within-lot variance was

$$s^2 = \frac{\sum_{i=4}^{7} \sum_{k=1}^{n_i} (\hat{a}_1 + \hat{b}_1 S_i - Y_{ik})^2}{\left(\sum_{i=4}^{7} n_i\right) - 2} = .034$$

and again, the between lot variance, σ_B^2 was for the sake of an example, taken to be 0.05. The lower tolerance boundary was calculated using the formula

$$L(S) = \hat{a}_1 + \hat{b}_1 S - k\,\hat{\sigma} = 8.09 - 0.0763S - k(0.290)$$

where $S > S_k$ and

$$\hat{\sigma} = \sqrt{S^2 + \sigma_B^2} = \sqrt{.034 + 0.05} = .290$$

and k is the 99/90 tolerance factor for the non-central t-distribution corresponding to

$$f = \left(\sum_{i=4}^{7} n_i\right) - 2$$

and

$$n = \cfrac{1}{\cfrac{1}{\sum_{i=4}^{7} n_i} + \cfrac{(S - \bar{S})^2}{\sum_{i=1}^{4} n_i (S_i - \bar{S})^2}}$$

(Note that n depends on S).

Analogous formulas were used to compute estimates of the regression line for stress levels at or below 30 ksi, and the corresponding lower tolerance boundary. The results obtained were

$$\hat{b}_2 = -\,0.165$$

$$\hat{a}_2 = 11.0$$

and $$L(S) = 11.0 - 0.165S - k\ 0.416$$

<u>Model 3</u> Least Squares Fit of Log Cycle Life vs Log Stress to a Regression Line

In this model it is assumed that

$$f(S) = a + b \log S,$$

and that the within-lot variance of log cycle life is homogeneous throughout the stress range tested. This assumption of equal variances could lead to the same problems as encountered in Model 1; however, this method has the advantage of providing explicit formulas, estimating f(S) and L(S), which hold throughout the stress range tested. Estimates of a and b were calculated analogously to the estimates of $\hat{a}_1$ and $\hat{b}_1$. The resulting estimate $\hat{f}(S)$, of f(S) was

$$\hat{f}(S) = 18.76 - 8.58 \log S$$

The within-lot estimate of variance was

$$s = 0.075,$$

and as before the between-lot variance was taken to be $\sigma_B^2 = 0.05$.

The resulting lower tolerance boundary was

$$L(S) = 18.76 - 8.58 \log S - k\ .353.$$

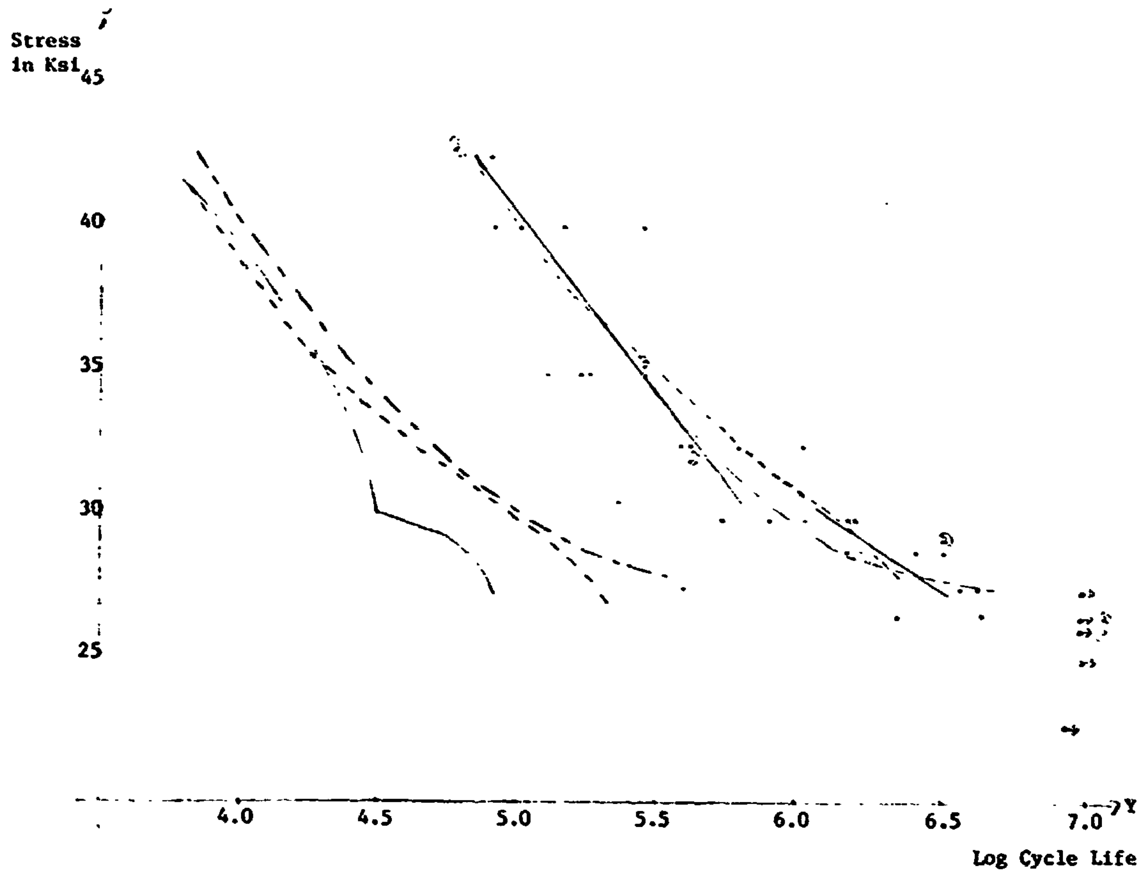


FIGURE 1

GRAPH OF FATIGUE DATA WITH ESTIMATED MEAN LOG CYCLE LIFE VS STRESS AND LOWER 99/90 TOLERANCE BOUNDARY FOR LOG CYCLE LIFE

Code: – — – — – Model 1: Fit sample mean log cycle life vs. stress, with corresponding lower tolerance boundary.

———— Model 2: Least squares fit of log cycle life vs. stress to two regression lines, with corresponding lower tolerance boundary.

- - - - - - Model 3: Least squares fit of log cycle life vs. log stress to a regression line, with corresponding lower tolerance boundary.

TABLE 1

FATIGUE DATA FOR 7039 AL

(Cycle life to failure or runout x 10^{-5} at 150°F,
Log cycle life to failure or runout at 150°F)

Stress	Log Stress						
42.5 ksi	1.628	Cycles	0.629	0.755	0.639	0.631	
		Log Cycles	4.80	4.88	4.81	4.80	
40	1.602	Cycles	1.13	1.441	2.8	0.789	0.998
		Log Cycles	5.05	5.16	5.45	4.90	5.00
35	1.544	Cycles	2.798	1.206	1.843	2.84	1.74
		Log Cycles	5.45	5.10	5.26	5.45	5.24
32.5	1.512	Cycles	4.007	10.218	4.112	4.071	
		Log Cycles	5.60	6.01	5.61	5.61	
30	1.477	Cycles	5.294	7.956	15.949	15.488	13.1
		Log Cycles	5.72	5.90	6.20	6.19	6.12
28.75	1.455	Cycles	15.5	33.8	34.	25.9	
		Log Cycles	6.19	6.53	6.53	6.41	
27.5	1.439	Cycles	$100^{\rightarrow}$	43.7	37.6	3.6	
		Log Cycles	$7^{\rightarrow}$	6.64	6.58	5.56	
26.5	1.423	Cycles	$100^{\rightarrow}$	46.5	$100^{\rightarrow}$	22.8	
		Log Cycles	$7^{\rightarrow}$	6.67	$7^{\rightarrow}$	6.36	
26	1.415	Cycles	$100^{\rightarrow}$	$100^{\rightarrow}$	$440^{\rightarrow}$		
		Log Cycles	$7^{\rightarrow}$	$7^{\rightarrow}$	$7.64^{\rightarrow}$		
25	1.398	Cycles	$100^{\rightarrow}$				
		Log Cycles	$7^{\rightarrow}$				
22	1.342	Cycles	$100^{\rightarrow}$				
		Log Cycles	$7^{\rightarrow}$				

TABLE 2

MODEL 1 ANALYSIS

Stress	Sample Size	D.f.	Sample Mean Log Cycle Life	Sample Variance (For within lot variation)	Assumed Between Lot Variance	Estimated Variance About the Mean	Tolerance Factor	Lower Tolerance Point
	n_i		$\overline{Y}_i$	S_i^2	$\hat{\sigma}_B^2$	$\hat{\sigma}_i^2$	k	$L(S_i)$
42.5	4	25	4.82	0.00113	0.05	0.051	3.50	3.87
40	5	25	5.11	0.0352	0.05	0.085	3.42	4.18
35	5	25	5.29	0.0246	0.05	0.075	3.42	4.37
32.5	4	25	5.71	0.0427	0.05	0.092	3.50	4.76
30	5	25	6.03	0.0439	0.05	0.094	3.42	5.11
28.75	4	25	6.42	0.0257	0.05	0.076	3.50	5.47
27.5	5	25	6.42	0.2895	0.05	0.340	3.42	5.50
26.5	4	--	6.84(median)	--	--	--	--	--

NOTE: Pooled estimate of std. dev. about the mean is $\sqrt{\frac{\sum_{i=1}^{7} (n_i - 1)\, s_i^2}{\sum_{i=1}^{7} (n_i - 1)} + \sigma_B^2} = .269$

TABLE 3

MODEL 2 ANALYSIS

Stress	n	D.f.	Mean Log Cycle Life	Sample Variance (within lots)	Assumed Var. Between Lots	Est. Var. About Mean	Tolerance Factor	Lower Tolerance Point $L(S_i)$
42.5	2	16	4.85	.034	0.05	$(.290)^2$	4.00	3.66
40	6	16	5.04	.034	0.05	$(.290)^2$	3.58	4.00
35	6	16	5.42	.034	0.05	$(.290)^2$	3.58	4.38
32.5	2	16	5.61	.034	0.05	$(.290)^2$	4.00	4.42
30	5	12	6.06	.123	0.05	$(.416)^2$	3.83	4.47
28.75	14	12	6.31	.123	0.05	$(.416)^2$	3.64	4.80
27.5	5	12	6.47	.123	0.05	$(.416)^2$	3.83	4.88
26.5	-	-	-	-	-	-	-	-

TABLE 4

MODEL 3 ANALYSIS

Stress	Sample Size n	D.f.	Mean Log Cycle Life	Sample Variance (within lots)	Assumed Var. Between Lots	Est. Var. About Means	Tolerance Factor	Lower Tolerance Point $L(S_i)$
42.5	8	30	4.78	.075	.05	$(0.353)^2$	3.24	3.63
40	13	30	5.00	.075	.05	$(0.353)^2$	3.14	3.89
35	28	30	5.50	.075	.05	$(0.353)^2$	3.05	4.42
32.5	31	30	5.78	.075	.05	$(0.353)^2$	3.04	4.71
30	21	30	6.08	.075	.05	$(0.353)^2$	3.08	4.99
28.75	16	30	6.23	.075	.05	$(0.353)^2$	3.11	5.13
27.5	12	30	6.40	.075	.05	$(0.353)^2$	3.15	5.29
26.5	-	-	-	-	-	-	-	-

TABLE 5

SUMMARY OF ANALYSIS

Stress	Mean Log Cycle Life Model 1	Model 2	Model 3	Lower Tolerance Boundary Model 1	Model 2	Model 3
42.5	4.82	4.85	4.78	3.87	3.66	3.63
40	5.11	5.04	5.00	4.18	4.00	3.89
35	5.29	5.42	5.50	4.37	4.38	4.42
32.5	5.71	5.61	5.78	4.76	4.42	4.71
30	6.03	6.06	6.08	5.11	4.47	4.99
28.75	6.42	6.31	6.23	5.47	4.80	5.13
27.5	6.42	6.47	6.40	5.50	4.88	5.29
26.5	6.84(median)	-	-	-	-	-

NERVA PROGRAM RELIABILITY PROCEDURE	NUMBER: R101-NRP-503	REVISION
	EFFECTIVE DATE:	CATEGORY III
STATISTICAL ANALYSIS OF MATERIALS TEST DATA	SUPERSEDES: NUMBER: DATE:	
	APPROVED BY:	

1.0 PURPOSE

Accurate information about material properties is necessary in the designing of an item to specified numerical reliability and in the maintaining of design reliability during subsequent fabrication. However, for a variety of reasons, different specimens of a given material do not have precisely the same characteristics, and this variability must be taken into account in test planning and reliability calculations. Sources of variability may include inherent non-homogeneity of the material, vendor-to-vendor differences, heat-to-heat (lot-to-lot) variations, etc. Testing a sample of specimens of a given material according to a properly designed plan provides partial information about the distribution of values of a parameter (e.g., tensile strength), and this information can be statistically analyzed to provide derived information such as lower tolerance limits (design allowables). (The test procedure itself introduces further variability, due to measurement error, test machine variation, operator error, etc., which is taken account of in the statistical analysis.)

The purpose of this procedure is to describe methods of materials test planning which are statistically efficient, and valid methods of analyzing the resulting data.

The requirement for this procedure is set forth in Data Item R-101, NERVA Reliability Program Plan.

2.0 APPLICABLE DOCUMENTS

2.1 Data Item R-101, NERVA Reliability Program Plan
2.2 SNPO-C-1, NERVA Program Structural Design Requirements
2.3 Data Item R-106, Reliability Test and Evaluation Plan
2.4 NRP-600, Statistical Distributions, Their Applications and Tables
2.5 NRP-601, Error in Assumption of Normality

3.0 POLICY

3.1 Materials properties data are an integral part of the design for reliability process during both the pre-design analytical activities and the post-design test and evaluation activities.

3.0 Policy (continued)

3.2 Proper interpretation and application of the methods described herein are essential for all engineers influencing the design and analysis of materials tests and utilizing the results of such tests.

4.0 <u>DEFINITIONS</u>

4.1 CONFIDENCE LEVEL

The confidence level attached to a statement based on sample data (e.g., that a certain proportion of a population lies above a calculated tolerance limit) is the probability that the statement is true.

4.2 <u>POWER</u>

The power of a test of a statistical hypothesis is the probability that the test will reject the hypothesis when it is false.

4.3 SIGNIFICANCE LEVEL

The significance level of a test of a statistical hypothesis is the probability that the test will reject the hypothesis when it is true.

4.4 TOLERANCE LIMIT

A 100P%/100γ% lower tolerance limit is a number, calculated from a sample drawn from a given population, which has the property that, with confidence level 100γ%, it lies below 100P% of the population.

5.0 <u>PROCEDURE</u>

5.1 THE NEED FOR STATISTICAL TREATMENT OF TEST DATA

Materials testing is undertaken to establish properties of materials for use in design and reliability assessment, to compare materials which are competitors for a given application, to examine the effects of varying a parameter such as temperature on the performance of the material, etc. If all specimens of a given material could be assumed to be identical with respect to the property of interest, if test conditions could be absolutely controlled and if measurements could be made with perfect accuracy, then we would need to test only one specimen to get the desired information, and statistical methods would be unnecessary. In fact, however, none of the above assumptions is realistic, and test results vary from specimen to specimen even though controllable test conditions do not change. The purpose of statistical

5.0 Procedure (continued)

design and analysis of materials test plans is to provide efficient and logical methods of extracting the desired information from the test data in the face of the obscuring effects of test variability, or "experimental error". Statistical methods cannot provide information about materials properties with certainty, but they can provide a numerical measure of the degree of uncertainty associated with the partial information contained in the test results.

5.2 EXPERIMENTAL ERROR

5.2.1 General

As explained previously, experimental variability, or "experimental error" as it is sometimes called, is the reason for employing statistical methods in the analysis of test data. If we liken the test data to a signal which contains the desired information about material properties, we can think of the experimental error as noise which is obscuring the clarity of the signal. The purposes of experimental design are to minimize this noise, or experimental error, and to provide a measure of it as a yardstick against which temperature or other effects may be compared. It is of interest to have some knowledge of the sources of variability, both as an aid to better understanding of the methodology of test planning and analysis, and as an indication of ways in which variability might be reduced.

5.2.2 Sources of Experimental Error

The chief sources of experimental error in materials test planning may be classified as follows:

5.2.2.1 Product Variability. Because of such factors as non-homogeneity of the composition of materials, random distribution of micro-cracks, and variations in fabrication processes (forging, rolling, etc.), specimens taken from the same "lot" (sheet, bar, forging, etc.) exhibit variations with respect to mechanical, thermal and physical properties. If specimens are taken from different lots, there may be additional variability due to lot differences, and this will be added to experimental error unless specifically accounted for in the test plan and the analysis. (If the lots themselves come from different "heats" or batches of parent material, this constitutes another source of experimental error unless separately accounted for.)

5.2.2.2 Technical Errors.

a. Non-reproducibility of a test condition - e.g., temperature.
b. Test-to-test variation in the operation of a single piece of test equipment.
c. Differences in performance of different test machines.
d. Operator error.
e. Operator differences.

5.0 Procedure (continued)

5.2.2.3 Measurement Errors. Error of measurement instruments in recording values of response variables, and error in reading measurement instruments.

5.2.2.4 Non-uniformity of Unsuspected, Ignored or Uncontrollable Conditions. Time-of-day or atmospheric conditions, for example, may not be taken account of as test variables. If they do in fact influence test results, they are a source of experimental error.

5.2.2.5 Data Processing Errors. Round-off errors, transcription errors, etc.

5.2.3 <u>Reducing Experimental Error</u>

Reducing experimental error increases the information available from a given number of tests, or reduces the number of tests required to deliver a given amount of information. In either case, the result is a reduction in cost of information, so on grounds of economy close attention should be paid to the sources of error. All test conditions which can be identified as potentially affecting test results should be taken into account. Instruments, test machines and operators need to be efficient, and the testing and recording operations should be carefully supervised. As far as possible testing should be done at one period of time and in one place, and should be done using as few machines and operators as possible.

5.2.4 <u>Systematic Error</u>

5.2.4.1 General. Though experimental error (noise) obscures the information in the signal and thus should be minimized, even more serious is systematic error, which can badly distort (bias) the test information. An example of systematic error is testing all specimens for a given set of test conditions at one time using one machine and one operator, and then testing all specimens for another set of test conditions at another time using another machine and another operator. The effect of changing test conditions is then inextricably mixed ("confounded") with time, machine and operator differences. Systematic errors of this kind can be avoided by introducing such factors as time, machines and operators explicitly into the analysis (blocking), or by spreading their influence impartially over the various test conditions (randomization). If it is possible, blocking is to be preferred to randomization for dealing with such factors as test machine differences, because it removes the source of variability altogether, and thus reduces experimental error. Randomization removes bias, but does not reduce the experimental error.

5.2.4.2 Blocking to Eliminate Systematic Error. A simple example of blocking is where there are 2 test machines, and 4 specimens to be tested at each test condition. Half of the total specimens are tested on one machine, 2 specimens at each test condition, and similarly half are tested on the other

5.0 Procedure (continued)

machine. This plan is called a randomized complete block design. More elaborate blocking schemes are available, and expert statistical assistance should be sought in planning a test incorporating blocking.

5.2.4.3 Randomization. Whether or not a test plan utilizes blocking to remove some of the extraneous variability, randomization is an absolutely essential element in the plan if the resulting information is to be valid and accurate. Randomization is necessary to remove any known sources of bias (such as test machine differences), to remove less visible bias (such as intentional or unintentional "selection" of specimens, as when all specimens tested at one condition were fabricated on one machine, and all tested at another condition were fabricated on another machine), and finally to ensure that the error variance is correctly estimated and that the probabilistic assumptions on which the analysis rests are reasonably well satisfied. Randomization is not an optional feature of a test plan; if it is not properly done, little confid e can be had in the validity of the results.

As an example of randomization, consider a plan where 16 specimens are obtained from each of 2 lots, and 4 of the 16 from a given lot are tested at each of 4 test conditions. Suppose the testing must be done at 2 different time periods. If the plan is blocked by periods, the test matrix is:

	LOT 1				LOT 2			
	TEST CONDITION				TEST CONDITION			
	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4
Period 1	2	2	2	2	2	2	2	2
Period 2	2	2	2	2	2	2	2	2

Randomization would be achieved by allocating the 16 specimens from each lot randomly to condition/period combinations (2 to each combination). (Random allocation can be achieved using tables of random numbers). It may be possible and desirable to randomize the order of testing within each period, by putting the 16 specimens into random order and testing them in that order. (Tables of random permutations can be used for this purpose). Or it may be necessary to test in random order in subgroups, e.g., the 4 specimens at a given test condition.

There may be constraints which, in some degree, prevent blocking and/or randomization. In the above example, it may be that all tests at conditions C_1 and C_2 must be done in period 1, and all at conditions C_3 and C_4 in period 2. In this case, confounding of condition effects with period effects occurs and should be recognized. But confounding should be avoided if at all possible, even if it involves some additional cost. In the above example, if all specimens from lot 1 were tested in period 1, and all from lot 2 in period 2, the result, if period effects are present, may be an erroneous estimate of lot effects, which may have serious and costly consequences (such as requiring further testing).

5.0 Procedure (continued)

It is emphasized again that proper randomization is vital to the securing of valid results from costly testing programs. Qualified personnel are needed for this aspect of materials test planning - it cannot be left to technicians. Moreover, proper supervision is necessary to ensure that the specified randomization is carried out.

5.3 THE NUMBER AND ALLOCATION OF SPECIMENS

5.3.1 Balance in Statistical Design

Given the objectives of a material test plan, a decision needs to be made on the number of specimens to be tested and their allocation among lots, test conditions, etc.

As a general rule, every effort should be made to ensure that the test design is completely balanced. By this is meant that an equal number of specimens should be tested for each combination of lot, test condition and block. (If the amount of testing required to achieve complete balance is prohibitive, more complex designs involving incomplete blocks, fractional factorials, etc., may be applicable. Such designs will not be discussed in this procedure - expert assistance should be sought if it is thought that one of these designs may be applicable.) If the design is not balanced, it may be difficult or impossible to satisfactorily analyze the data, and in any case the results of the analysis are likely to be less accurate and informative than those from a balanced design with an equal number of observations. One aspect of these problems is confounding of effects. As an extreme example of confounding, consider a design with 2 lots and 2 test conditions. If all specimens from one lot are tested at one condition, and all specimens from the other lot are tested at the other condition (extreme imbalance), then there is no way of separating lot effect from test condition effect - the two are completely confounded. Partial confounding occurs whenever there is imbalance in a design.

Achieving a balanced design requires more than simply specifying equal numbers of observations in cells. Extra specimens may be needed in case of "bad" tests, accidental destruction, etc., and adequate supervision of the testing process is required so that the specified number of usable observations is achieved.

5.3.2 Determination of the Numbers of Observations

The total number of observations required in a test plan in general depends on two factors: first, the kind and precision of the information desired; second, certain aspects of the underlying distribution (e.g., the error variance). While we may be able to specify in advance the nature and precision of the information we want, we ordinarily have little knowledge of the distribution of the material property in question. Consequently, sample size determinations can usually only be rough

5.0 Procedure (continued)

approximations. We give two simple examples to illustrate the reasoning involved:

Example 1: Let X represent the "strength" of a randomly chosen specimen of a certain material, and suppose X is normally distributed with mean μ and variance σ^2. Suppose a part succeeds if X exceeds a, so that the reliability of the part is $R = P(X \geq a)$. Suppose $(\mu - a)/\sigma = 4.75$, so that $R = .9_6$. We would like to be able to demonstrate, at confidence level 95%, that the reliability is $.9_6$ or more. How many specimens should be tested?

A lower 95% confidence limit on R is obtained by putting $k = \frac{\bar{X} - a}{s}$ (where $\bar{X}$ and s are respectively the sample mean and standard deviation), and looking up the probability corresponding to k in normal tolerance limit tables. Since $\frac{\bar{X} - a}{s}$ will exceed 4.75 in about ½ of all samples, we have about a 50% chance of demonstrating a probability at least as large as that given in the table. Sample sizes for various reliabilities are given below:

Sample Size	Reliability
7	$.9_2$
14	$.9_3$
33	$.9_4$
144	$.9_5$

Thus the answer to the original question is that we would need a sample of 144 observations to have about a 50% change of demonstrating a reliability of $.9_5$ or better with 95% confidence. (If we wanted a better chance, say 90%, of demonstrating $.9_5$, a much bigger sample would be necessary.)

Example 2: Suppose the variability of specimens with respect to a certain property is the sum of two components, a lot-to-lot variance (σ_δ^2) and a variance within lots (σ_ϵ^2). If a sample of n specimens is taken from each of I lots, then an approximate lower 99/95 lower tolerance limit (for use in design) is given by $\bar{X} - k\, s_T$, where $\bar{X}$ is the grand mean of the I n observations, s_T is an estimate of $\sigma_T = \sqrt{\sigma_\delta^2 + \sigma_\epsilon^2}$, and k is a tolerance factor found according to the method of Section 5.4.5.4. Suppose we want to obtain a k of about 3; how many lots (I) and observations per lot (n) should we test? The method of Section 5.4.5.4 can be shown to imply that to get a k of about 3 we need

$$f = \frac{n^2 I\,(I-1)\,(1+R)^2}{(I-1)\,(n-1) + I\,(1 + nR)^2} = 35, \text{ where } R = \sigma_\delta^2 / \sigma_\epsilon^2 .$$

Suppose R = 1/4. Then if we take I = 2, we can never achieve f = 35, no matter how large n is. If n = 100 (so the total number of observations is 200), we get f = 21.5 and k = 3.23. However, if we sample 4 lots (I = 4), we need only 18 observations per lot (a total of 72) to get f = 35 and thus k = 3.

5.0 Procedure (continued)

Before testing has been done, we may have only a rough notion of the precision we require, and we are likely to have only very crude estimates of the degree of variability of the distribution involved. However, calculations like those illustrated above may be helpful in deciding on an appropriate sample size, or on a reasonable allocation of total specimens as regards number of lots and specimens per lot. If possible, all the testing for a given material property should be done at one time. Although it may be possible to do more testing after the initial results reveal that more data are necessary to achieve the required objectives, there are good reasons for attempting to avoid this testing by stages. The time lapse involved may lead to inflation of lot variability because of possible differences in testing machines, operators, testing techniques, etc.

5.3.3 Importance of the Numbers of Lots Tested

The minimum number of lots on which an estimate of the lot-to-lot variance can be based is 2. Example 2 above illustrates the fact that a sample of 2 lots contains very little information about lot variability. Unless it can be confidently judged (before testing) that the lot-to-lot variance is small relative to the within-lot variance, it is risky to test only 2 lots. Ordinarily, it is better, given a fixed total number of observations, to test more than 2 lots and a correspondingly smaller number of specimens per lot.

5.4 STATISTICAL ANALYSIS OF TEST DATA

5.4.1 General

In this section, the statistical analysis of a variety of test designs is described in detail. The list of designs discussed is by no means exhaustive, but should cover a large proportion of materials test plans. Statistical assistance should be sought in planning and/or analyzing designs not specifically covered here.

The important topic of planning the experiment in the sense of choosing the test conditions and variables, and the levels of the test variables, is not covered in this procedure. These choices are determined by the information required from the experiment, which, in turn, depends on the use to be made of the information.

5.4.2 Fixed and Random Effects

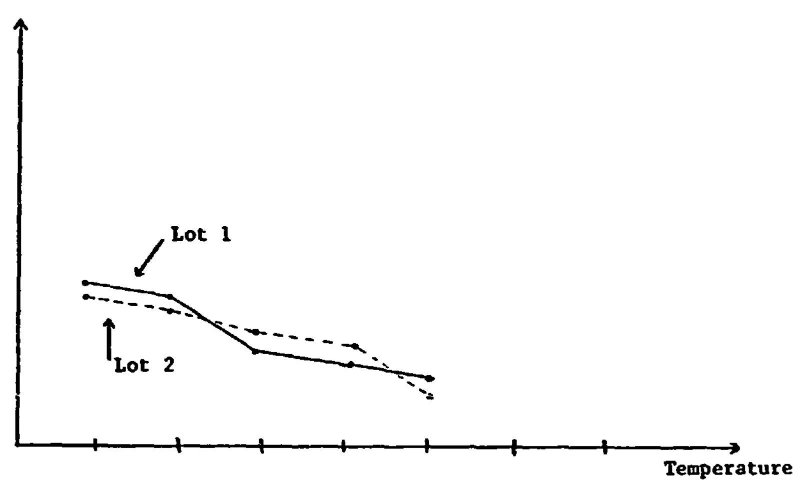
An important distinction needs to be made between "fixed effects" factors (test variables) and "random effects" factors. A factor is a fixed effects factor if its levels are set, or fixed (e.g., temperature levels); on the other hand, it is a random effects factor if its levels are

5.0 Procedure (continued)

supposedto be chosen at random from a whole population of possible "levels" (e.g., production lots of a material.) Whether a factor is fixed or random has a radical effect on the analysis of the data. Ordinarily in materials testing the only random effects factor will be lots (bars, sheets, forgings, etc., which may be from the same or different heats.) Typical fixed effects factors are temperature, directionality of measurement, and radiation level. If material is being obtained for testing from different vendors, then vendors will be either a random or fixed effects factor, depending on whether the vendors represented in the test are, or are not, considered to be a random sample from a "population" of vendors (this decision would depend on whether or not future material procurement would be restricted to the vendors represented in the sample).

5.4.3 Interactions

In the designs discussed, there is either 0 or 1 random effects factor (lots), and 0, 1 or 2 fixed effects factors (exemplified by temperature and direction). Interactions between fixed effects factors are specifically accounted for in the analyses presented, but interactions between lots and fixed effects factors are assumed to be zero. (Interactions are present if the differential effects of changing the levels of one factor are not the same for all levels of the other factor.) Though in many cases that may be a reasonable assumption, the data should always be inspected to see if such interactions are apparent. This can be done by plotting the means at different temperatures for each lot, joining the points and seeing whether the resulting curves are roughly parallel, as they are in the following graph:



If the curves are not approximately parallel, the implication is that lot-temperature interactions may be present. In this case, statistical assistance should be sought in performing further analysis.

5.0 Procedure (continued)

5.4.4 Interpretation of Lower Tolerance Limits

One of the chief purposes of materials testing is to derive lower tolerance limits (design allowables). The methods described in this procedure for deriving these limits carry some implicit assumptions about future material procurement. The assumption of the derivation is that we are sampling from a hypothetical population of material, which consists of all material which might be produced by the vendor(s) represented in the sample, using processes, raw materials, etc., essentially similar to those used in producing the sampled material. We derive a number from the sample which with a specified confidence level is a lower bound with respect to the property in question on 99% (say) of the population. Using this number in design, etc., implies that when we purchase material for fabrication, we expect to obtain a random sample from the same hypothetical population as that sampled at the time of testing. There are several reasons why this may be an unrealistic assumption. First, we may not know whether purchases for fabrication will be from the vendor who supplied the test material. Second, even if the vendor is the same, his manufacturing processes, etc., may have changed in the time interval between testing and fabrication. Third, no account is taken of the possible effects of acceptance testing.

No attempt is made here to suggest alternative techniques to deal with these considerations, but their existence needs to be recognized in interpreting the results of analyses given in this procedure.

5.4.5 Analysis of Several Experimental Layouts

The statistical analysis of several experimental layouts is presented below. This analysis includes:

(a) An estimate of the mean response for each given test condition;
(b) An estimate of the within-lot variance of the response and, when more than one lot of specimens is to be tested, an estimate of the variance of the response which takes lot-to-lot variation into account;
(c) Tests of various hypotheses;
(d) A 100P%/100Y% lower tolerance limit for the response.

The analysis of variance calculations can be performed using a computer library program, for example "ANVA1" or ANVA5" in the G.E. Mark II Time-Sharing Service.

5.4.5.1 One Lot, One Test Condition. Assume that one lot of specimens is to be tested for the response to one test condition. The model equation is: $Y_i = \mu + \epsilon_i$; $i = 1, 2, \ldots n$, where Y_i is the response, μ is the mean response to the given test condition and ϵ_i is a term representing the "experimental error". In Case 1 and all the other cases, "ϵ" with any number of subscripts represents the experimental error; it will always be assumed that $\epsilon \sim N(o, \sigma_\epsilon^2)$, and that the ϵ's are mutually

5.0 Procedure (continued)

independent (σ_ϵ^2 is the variance of the experimental error).

An estimate of μ is $\bar{Y}. = \sum_{i=1}^{n} Y_i / n$.

An estimate of σ_ϵ^2 is $s^2 = \sum_{i=1}^{n} (Y_i - \bar{Y}.)^2 / (n - 1)$.

(Note: In all cases replacing subscripts of a variable by dots indicates summing that variable over the ranges of values of the subscripts which are replaced by dots. For example: $Y. = \sum_{i=1}^{n} Y_i$. Further, a bar placed over a dotted variable indicates that the dotted variable is divided by the total number of observations involved in the summation. For example: $\bar{Y}. = \sum_{i=1}^{n} Y_i/n$.)

A lower 100P%/100Y% tolerance limit is $\bar{Y}. - ks$, where k is the appropriate tolerance factor (k is obtained from the Sandia Corporation Tables, Reference 2, Table 2, pp 27 - 105).

5.4.5.2 One Lot, One-way Classification of Test Conditions. Assume that one lot of specimens is to be tested for response to a one-way classification of test conditions (say temperature). The model is: $Y_{ij} = \mu + T_i + \epsilon_{ij}$; $i = 1, \ldots, I$ and $j = 1, \ldots, n_i$. T_i is the effect of the i^{th} test condition, while μ is the overall mean response for all test conditions.

Analysis of Variance (ANOVA)

Source of Variation (Source)	Degrees of Freedom (d.f.)	Sum of Squares (S.S.)	Mean Square (M.S.)	Expected Mean Square (E.M.S.)
Between test conditions	I-1	$SSB = \sum_{i=1}^{I} n_i(\bar{Y}_{i.} - \bar{Y}..)^2$	MSB = SSB/(I-1)	$\sigma_\epsilon^2 + \sum_{i=1}^{I} n_i T_i^2/(I-1)$
Error	$f = \sum_{i=1}^{I} (n_i - 1)$	$SSE = \sum_{i=1}^{I} \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_i.)^2$	MSE = SSE/f	σ_ϵ^2
Total	$\left(\sum_{i=1}^{I} n_i\right) - 1$	$SST = \sum_{i=1}^{I} \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}..)^2$		

An estimate of $\mu + T_i$ (the mean response at test condition i) is $\bar{Y}_i. = \sum_{j=1}^{n_i} Y_{ij}/n_i$.

An estimate of σ_ϵ^2 is s^2 = MSE.

5.0 Procedure (continued)

To determine whether there are significant differences in response for different test conditions, the hypothesis that $T_i = 0$ for $i = 1, \dots, I$ is tested at the 100α% significance level (e.g., $\alpha = .05$). Let $F = MSB/MSE$, and let

$F_{1-\alpha;\ I-1,\ f}$ = the upper 100 $(1-\alpha)$% point of the F-distribution with I-1 and f degrees of freedom

(cf. NRP-600, Table 5.4.10, p 174 ff, where $df_1 = I-1$ and $df_2 = f$). If $F > F_{1-\alpha;\ I-1,\ f}$, then the hypothesis that $T_i = 0$ for $i = 1, \dots, I$ is rejected, and it can be concluded that, at the 100α% significance level, there are significant differences in response among test conditions. If $F \le F_{1-\alpha;\ I-1,\ f}$ then it can be concluded that at the 100α% significance level there are no significant differences in response among test conditions.

To test whether there is a significant difference in response between test condition i and test condition j (for some particular i and j) at the 100α% significance level, let

$t = \dfrac{|\overline{Y}_{i.} - \overline{Y}_{j.}|}{s\sqrt{\frac{1}{n_i} + \frac{1}{n_j}}}$, and let $t_{1-\alpha/2;\ f}$ = the upper 100 $(1-\alpha/2)$% point of the t-distribution with f degrees of freedom

(cf. NRP-600, Table 5.4.9, p 171 ff., where df = f). If $t > t_{1-\alpha/2;\ f}$ it is to be concluded that at the 100α% significance level, the mean response at test condition i significantly differs from the mean response at test condition j. If $t \le t_{1-\alpha/2;\ f}$ it is concluded that the mean response at test condition i does not significantly differ from the mean response at test condition j, at the 100α% significance level.

Let k be the $100P\%/100\gamma\%$ tolerance factor obtained from the Sandia Corporation Tables (Table 4, Reference 2, pp 163 - 252; in this table, take $n = n_i$ and $f = \sum_{i=1}^{I} (n_i - 1)$). If appropriate values are not tabulated, use the computer program "TFAC**", to compute k with $m = n_i$, $f = \sum_{i=1}^{I} (n_i - 1)$. (The use of "TFAC**" is explained in Appendix 1.) The lower tolerance limit for the response at test condition i is then $\overline{Y}_{i.} - ks$.

5.4.5.3 One Lot, Two-way Classification of Test Conditions. Assume that one lot of specimens is to be tested for the response to two different kinds of test conditions (i.e., a two-way classification of test conditions, say temperature and direction). The model is $Y_{ijk} = \mu + T_i + D_j + (TD)_{ij} + \epsilon_{ijk}$, where $i = 1, \dots, I$, $j = 1, \dots, J$, and $k = 1, \dots, n$. μ is the overall mean response for all test conditions. T_i is the effect of temperature i, D_j is the effect of direction j and $(TD)_{ij}$ is the temperature-direction interaction at test condition ij.

5.0 Procedure (continued)

ANOVA

Source	d.f.	S.S.	M.S.	E.M.S.
Between Temperatures	I-1	$SSTe = Jn \sum_{i=1}^{I} (\bar{Y}_{i..} - \bar{Y}_{...})^2$	SSTe/(I-1)	$\sigma_\epsilon^2 + \frac{Jn \sum_{i=1}^{I} T_i^2}{I-1}$
Between Directions	J-1	$SSD = In \sum_{j=1}^{J} (\bar{Y}_{.j.} - \bar{Y}_{...})^2$	SSD/(J-1)	$\sigma_\epsilon^2 + \frac{In \sum_{j=1}^{J} D_j^2}{J-1}$
Temperature-Direction Interaction	(I-1)(J-1)	$SSI = n \sum_{i=1}^{I} \sum_{j=1}^{J} (\bar{Y}_{ij.} - \bar{Y}_{i..} - \bar{Y}_{.j.} + \bar{Y}_{...})^2$	SSI/(I-1)(J-1)	$\sigma_\epsilon^2 + \frac{n \sum_{i=1}^{I} \sum_{j=1}^{J} (TD)^2_{ij}}{(I-1)(J-1)}$
Error	IJ(n-1)	$SSE = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{n} (Y_{ijk} - \bar{Y}_{ij.})^2$	SSE/IJ(n-1)	σ_ϵ^2
Total	IJn-1	$\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{n} (Y_{ijk} - \bar{Y}_{...})^2$		

An estimate of the mean response at test condition ij $(\mu + T_i + D_j + (TD)_{ij})$ is $\bar{Y}_{ij.} = \sum_{k=1}^{n} Y_{ijk} / n$.

An estimate of the error variance, σ_ϵ^2, is $s^2 = MSE = SSE/ IJ(n-1)$.

To test whether there is a significant temperature effect, compute $F_{Te} = [SSTe/(I-1)]/s^2$, and conclude that there is a significant temperature effect at significance level $100\alpha\%$ if $F_{Te} > F_{1-\alpha;\, I-1,\, IJ(n-1)}$.

5.0 Procedure (continued)

Similarly, compute $F_D = [SSD / (J-1)]/s^2$ and conclude that there is a significant direction effect at significance level 100α% if $F_D > F_{1-\alpha;\ J-1,\ IJ(n-1)}$.

The existence of interaction between temperature and direction can be tested by computing $F_I = [SSI / (I-1)(J-1)]/s^2$ and concluding that there is significant interaction at the 100α% significance level if $F_I > F_{1-\alpha;\ (I-1)(J-1),\ IJ(n-1)}$.

To test whether the mean response of the i^{th} temperature equals the mean response of the j^{th} temperature, compute

$$t_{Te} = \frac{|\bar{Y}_{i..} - \bar{Y}_{j..}|}{s\sqrt{2/(Jn)}}$$

and conclude that the mean effect of temperature i differs from the mean effect of temperature j at the 100α% significance level if $t_{Te} > t_{1-\alpha/2;IJ(n-1)}$.

Similarly, one can compute

$$t_D = \frac{|\bar{Y}_{.i.} - \bar{Y}_{.j.}|}{s\sqrt{2/(In)}}$$

and conclude that the mean response for direction i significantly differs from the mean response for direction j at significance level 100α% if $t_D > t_{1-\alpha/2;\ IJ(n-1)}$.

The lower 100P%/100α% tolerance limit for the response at test condition ij is $\bar{Y}_{ij.}$ - ks , where k is obtained from Table 4 of the Sandia Corporation Tables (n = n, f = IJ(n-1)). If appropriate values are not tabulated, use the computer program "TFAC**" to compute k, with m = n and f = IJ(n-1).

5.4.5.4 I Lots, One Test Condition. Assume that I lots of specimens are to be tested at one test condition. The model is

$$Y_{ij} = \mu + \delta_i + \epsilon_{ij} ,$$

where i = 1, ... , I and j = 1, ... , n_i . It is assumed in this case, and all of the following cases, that the δ_i's are independent of each other and of the ϵ's and that $\delta_i \sim N(o, \sigma_\delta^2)$ for i = 1,2, ..., I . (δ_i is the random effect of the i^{th} lot.)

5.0 Procedure (continued)

ANOVA

Source	d.f.	S.S.	M.S.	E.M.S.
Between Lots	$I-1$	$SSL = \sum_{i=1}^{I} n_i (\bar{Y}_{i.} - \bar{Y}_{..})^2$	$MSL = SSL/(I-1)$	$\sigma_\epsilon^2 + n_o\sigma_\delta^2$
Within Lots (error)	$f = \sum_{i=1}^{I} (n_i - 1)$	$SSE = \sum_{i=1}^{I} \sum_{j=1}^{ni} (Y_{ij} - \bar{Y}_{i.})^2$	$MSE = SSE/f$	σ_ϵ^2
Total	$\left(\sum_{i=1}^{I} n_i\right) - 1$	$\sum_{i=1}^{I} \sum_{j=1}^{ni} (Y_{ij} - \bar{Y}_{..})^2$		

In the above ANOVA,

$$n_o = \frac{\left(\sum_{i=1}^{I} n_i\right)^2 - \sum_{i=1}^{I} n_i^2}{(I-1)\sum_{i=1}^{I} n_i} .$$

The estimated mean response to the given test condition is $\bar{Y}_{..}$.

The estimate of within-lot (error) variance is MSE.

Lot-to-lot variance is estimated by $(MSL - MSE)/n_o$.

One may test whether the lot-to-lot variance is significantly greater than zero by using an F-test; this test is highly sensitive to departures from normality and hence needs to be interpreted with caution. To perform this F-test, calculate $F_L = \frac{MSL}{MSE}$ and conclude that the lot-to-lot variance is significant at the 100α% significance level if $F_L > F_{1-\alpha;\, I-1,\, f}$.

The lower 100P%/100γ% tolerance limit for the response is given by $\bar{Y}_{..} - ks_T$, where

$$s_T = \sqrt{s_\delta^2 + s_\epsilon^2} = \sqrt{\frac{MSL}{n_o} + \frac{n_o - 1}{n_o} MSE}$$

(s_T is the estimated variance of the response taking lot-to-lot variation into account), and k is the 100P%/100γ% tolerance factor computed from the computer program "TFAC**" using values of m and f calculated with the following formulas:

5.0 Procedure (continued)

$$f = [s_T^2]^2 \Big/ \left[\frac{\frac{1}{n_o^2} MSL^2}{I-1} + \frac{\left(\frac{n_o - 1}{n_o}\right)^2 MSE^2}{\sum_{i=1}^{I} (n_i - 1)} \right]$$

(f is the approximate degrees of freedom for s_T^2);

$$m = \frac{\frac{1}{n_o}[MSL + (n_o - 1) MSE]}{\frac{1}{\sum_{i=1}^{I} n_i} \left[\frac{(I-1)(MSL-MSE)}{\left(\sum_{i=1}^{I} n_i\right)^2 \Big/ \sum_{i=1}^{I} n_i^2 - 1} + MSE \right]} .$$

However if MSL < MSE (which would result in a negative estimate of lot-to-lot variance), the 100P%/100Y% tolerance limit is given by $\overline{Y}_{..}$ - kMSE, where k is obtained from "TFAC**" with

$$m = \sum_{i=1}^{I} n_i \text{ and } f = \sum_{i=1}^{I} (n_i - 1) .$$

5.4.5.5 I Lots, One-Way Classification of Test Conditions. Assume that I lots of specimens are tested for response to J test conditions (say temperature levels), and that there is no interaction between lots and temperatures. The model is $Y_{ijk} = \mu + \delta_i + T_j + \epsilon_{ijk}$, where μ, δ_i, T_j, ϵ_{ijk} are as described above.

A N O V A

Source	d.f.	S.S.	M.S.	E.M.S.
Between Lots	I-1	$SSL = Jn \sum_{i=1}^{I} (\overline{Y}_{i..} - \overline{Y}_{...})^2$	$MSL = \frac{SSL}{I-1}$	$\sigma_\epsilon^2 + Jn\,\sigma_\delta^2$
Between Temperatures	J-1	$SSTe = In \sum_{j=1}^{J} (\overline{Y}_{.j.} - \overline{Y}_{...})^2$	$MSTe = \frac{SSTe}{J-1}$	$\sigma_\epsilon^2 + In \frac{\sum_{j=1}^{J} T_j^2}{J-1}$
Error	f = IJn-I-J+1	SSE = SST-SSL-SSTe	$MSE = \frac{SSE}{f}$	σ_ϵ^2
Total	IJn-1	$SST = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{n} (Y_{ijk} - \overline{Y}_{...})^2$		

5.0 Procedure (continued)

The estimate of the mean response at test condition j is $\bar{Y}_{.j.}$.

The estimate of error variance is MSE.

The estimate of lot-to-lot variance is $\frac{MSL-MSE}{Jn}$.

The estimated variance of the response at a given test condition is

$$s_T^2 = \frac{MSL}{Jn} + \frac{Jn-1}{Jn} MSE .$$

To test whether the lot-to-lot variance is significant, compute $F_L = MSL/MSE$, and conclude the lot-to-lot variance is significant at level $100\alpha\%$ if $F_L > F_{1-\alpha;\ I-1,f}$ (this test is sensitive to non-normality).

The test for temperature effect is accomplished by computing $F_{Te} = MSTe/MSE$ and concluding that there is a significiant temperature effect at significance level $100\alpha\%$ if $F_{Te} > F_{1-\alpha;J-1,f}$.

One can test for the difference in response between temperature i and temperature j by computing

$$t = \frac{|\bar{Y}_{.i.} - \bar{Y}_{.j.}|}{\sqrt{\frac{2\ MSE}{In}}}$$

and concluding that, at significance level $100\alpha\%$, temperature i differs from temperature j if $t > t_{1-\alpha/2;f}$.

The lower tolerance limit for the response at temperature j is given by $\bar{Y}_{.j.} - ks_T$, where k is the one-sided 100P%/100γ% tolerance factor computed using "TFAC**" with

$$f = [s_T^2]^2 \Bigg/ \left\{ \left(\frac{MSL}{Jn}\right)^2 \Big/ (I-1) + \left(\frac{(Jn-1)MSE}{Jn}\right)^2 \Big/ [IJn-J-I+1] \right\}$$

and

$$m = \frac{s_T^2}{\frac{1}{In}\left[MSE + \frac{MSL-MSE}{J}\right]} .$$

5.0 Procedure (continued)

If MSL < MSE, the tolerance limit is $\bar{Y}_{.j.}$ - k MSE, where k is obtained from "TFAC**" with m = In and f = IJn-I-J+1.

5.4.5.6 I Lots, Two-way Classification of Test Conditions. Assume that there are two-way classification of test conditions (say temperature and direction) with I lots of specimens. Assume, further, that the temperature-lot, direction-lot and temperature-direction-lot interactions are zero. The model is

$$Y_{ijkh} = \mu + \delta_i + T_j + D_k + (TD)_{jk} + \varepsilon_{ijkh} .$$

5.0 Procedure (continued)

A N O V A

Source	d.f.	S.S.	M.S.	E.M.S.
Between Temperatures	J-1	$SSTe = IKn \sum_{j=1}^{J} (\overline{Y}_{.j..} - \overline{Y}_{....})^2$	MSTe = SSTe / (J-1)	$\sigma_\epsilon^2 + IKn \sum_{j=1}^{J} T_j^2 / (J-1)$
Between Directions	K-1	$SSD = IJn \sum_{k=1}^{K} (\overline{Y}_{..k.} - \overline{Y}_{....})^2$	MSD = SSD/(K-1)	$\sigma_\epsilon^2 + IJn \sum_{k=1}^{K} D_j^2 \ (K-1)$
Temperature-Direction Interaction	(J-1)(K-1)	$SSI = In \sum_{j=1}^{J} \sum_{k=1}^{K} (\overline{Y}_{.jk.} - \overline{Y}_{.j..} - \overline{Y}_{..k.} + \overline{Y}_{....})^2$	MSI=SSI/(J-1)(K-1)	$\sigma_\epsilon^2 + \frac{In \sum_{j=1}^{J} \sum_{k=1}^{K} (TD)^2_{jk}}{(J-1)(K-1)}$
Between Lots	I-1	$SSL = JKn \sum_{i=1}^{I} (\overline{Y}_{i...} - \overline{Y}_{....})^2$	MSL = SSL/(I-1)	$\sigma_\epsilon^2 + JKn\,\sigma_\delta^2$
Error	f = IJKn - I - JK + 1	SSE = SST - SSTe - SSD - SSI - SSL	MSE = SSE/f	σ_ϵ^2
Total	IJKn-1	$SST = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{h=1}^{n} (Y_{ijkh} - \overline{Y}_{....})^2$		

5.0 Procedure (continued)

The estimated mean response at test condition jk is $\bar{Y}_{.jk.}$.

The estimates of σ_ϵ^2 and σ_δ^2 are respectively MSE and $\frac{MSL-MSE}{JKn}$.

The estimated varience of the response at a given test condition is

$$s_T^2 = \frac{MSL}{JKn} + \frac{JKn-1}{JKn} MSE \;.$$

To test for temperature effects, for direction effects, for interaction between temperature and direction, or for lot-to-lot variance, compute $F = MS/MSE$ where MS is (respectively) MSTe, MSD, MSI or MSL. Conclude that the effect tested is significant at level $100\gamma\%$ if the corresponding F is greater than $F_{1-\gamma;h,f}$ where h is the degrees of freedom of the S.S. for the effect tested (e.g., if the temperature effect were tested, h would be J-1).

To test the difference between response at temperature i and response at temperature j, compute

$$t_{Te} = \frac{|\bar{Y}_{.i..} - \bar{Y}_{.j..}|}{\sqrt{MSE\left(\frac{2}{IKn}\right)}}$$

and conclude that the difference is significant at level $100\alpha\%$ if $t_{Te} > t_{1-\alpha/2;f}$.

Similarly test for a difference between direction i and direction j by computing

$$t_D = \frac{|\bar{Y}_{..i.} - \bar{Y}_{..j.}|}{\sqrt{MSE\left(\frac{2}{JKn}\right)}}$$

and concluding that the difference is significant at level $100\alpha\%$ if $t_D > t_{1-\alpha/2;f}$.

The lower $100P\%/100\gamma\%$ tolerance limit for the response at test condition jk is $Y_{.jk.} - k\, s_T$, where k is the tolerance factor computer by the "TFAC**" program with

$$f = [s_T^2]^2 \Big/ \left\{ \left(\frac{MSL}{JKn}\right)^2 \Big/ (I-1) + \left(\frac{(JKn-1)MSE}{JKn}\right)^2 \Big/ f \right\},$$

and

$$m = s_T^2 \Big/ \left\{ \frac{1}{In} \left[MSE + \frac{MSL-MSE}{JK} \right] \right\}.$$

If MSL < MSE, the tolerance limit is $\bar{Y}_{.jk.} - k$ MSE, where k is obtained from "TFAC**" with $m = In$ and $f = IJKn-I-JK+1$.

5.0 Procedure (continued)

5.5 ANALYSIS OF UNBALANCED DESIGNS

Apart from one-way classifications, all of the analyses discussed in Section 5.4.5 refer to balanced designs, that is, designs in which there are equal numbers of observations for lot-test condition combinations. If complete balance is not achieved, the methods of Section 5.4.5 cannot be directly applied. There are various methods which might be applicable in analyzing an unbalanced layout, such as missing-value techniques and multiple regression methods. But each case needs separate consideration, and statisrical assistance should be sought in analyzing unbalanced test data. Cost of analysis, as well as strength of information derived, is another consideration pointing to the desirability of achieving balance in test data.

5.6 THE ASSUMPTION OF NORMALITY

The models of Section 5.4.5 assume that both lot effects and experimental errors are normally distributed. For example, in the case of I lots and a one-way classification of test conditions (assuming zero interactions between lots and test conditions), the model equation is

$$Y_{ijk} = \mu + \delta_i + T_j + \epsilon_{ijk} ,$$

and we assume that

$$\delta_i \sim N(o,\sigma_\delta^2) \quad \text{and} \quad \epsilon_{ijk} \sim N(o,\sigma_\epsilon^2),$$

all of these random variables (δ's and ϵ's) being independent. The effects of departures from the normality assumption are discussed in NRP-601, "Error in Assumption of Normality". Bri[illegible] tests for the existence of test condition effects are not seriously disturbed by non-normality, but t[illegible] [illegible]out variances (e.g., the test that $\sigma_\delta^2 = 0$; the test for homogeneity of error variances) can be badly upse[illegible] if the distributions are not approximately normal. More is said about tests for homogeneity of variances in Section 5.7.2.

As far as tolerance limits are concerned, severe non-normality could undoubtedly have a serious effect on the results. Ordinarily, however, material property distributions can probably be expected to be not too far removed from the normal, particularly if the coefficient of variation is small. Thus the tolerance limit determinations in most cases are probably not far wrong, provided the other assumptions of the model hold.

If it is suspected that the data are not normal, then the tests or graphical techniques given in NRP-600 "Statistical Distributions, Their Applications, and Tables", or in NRP-601, might be applied, and perhaps a normalizing transformation might be considered. Ordinarily, however, there will be too few observations for the successful application of such techniques.

5.0 Procedure (continued)

Sometimes a variance-stabilizing transformation (see Section 5.7.5.1) has the happy effect of making the distributions more normal, but ordinarily, for materials distributions with small coefficients of variation, such transformations will not have much effect on normality, either favorable or unfavorable. On the other hand, if cell variances are approximately equal, an attempt to apply a normalizing transformation may cause serious inequality of variances.

5.7 UNEQUAL ERROR VARIANCES

5.7.1 General

All of the models discussed in Section 5.4.5 assume that the error variances are equal for all lot-test condition combinations. For example, in the case of I lots and a one-way test condition classification, the model equation is (assuming no interactions between lots and test conditions)

$Y_{ijk} = \mu + \delta_i + T_j + \epsilon_{ijk}$, and the assumption is made that $\epsilon_{ijk} \sim N(o,\sigma_\epsilon^2)$, so that

the error variance is assumed to be the same (σ_ϵ^2) for all lot-test condition combinations ij.

5.7.2 Testing for Variance Homogeneity

If the test design is balanced, then inequality of within cell variances does not seriously affect such tests as the F-test for existence of test condition effects (see Scheffé, Ch. 10, Reference 3). However tolerance limits are concerned with the behavior of a material at a given test condition, and need to be calculated using an estimate of variance at that test condition. Consequently, if the within cell variances are in fact unequal, then using the pooled estimate of within variance as described in Section 5.4.5 may lead to substantial error. Because of this, it is advisable to check the assumption of equal variances by performing a statistical test for homogeneity of variances. Provided the total number of observations is not too small (about 50 or more), the jackknife test, which is described in NRP-601, "Error in Assumption of Normality", is recommended for this purpose. This test is much less sensitive to non-normality of the data than the usual normal theory tests (Bartlett's test and Box's modification of Bartlett's test). If the total number of observations is less than 50 then it is recommended that Box's test be used.

5.7.3 Box's Test for Homogeneity

Box's test is performed as follows:

5.7.1 Suppose there are k cells, with n_i observations in the i^{th} cell. Let X_{ij} be the j^{th} observation in the i^{th} cell. Compute

5.0 Procedure (continued)

$$s_i^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_{i.})^2 , \qquad \text{where} \quad \bar{X}_{i.} = \frac{1}{n_i} \sum_{j=1}^{ni} X_{ij} .$$

5.7.3.2 Let $N = \sum_{i=1}^{k} n_i$. Compute

$$M = (N-k) \log \left[\sum_{i=1}^{k} (n_i - 1) s_i^2 \Big/ (N-k) \right] - \sum_{i=1}^{k} (n_i - 1) \log s_i^2$$

$$A = \frac{1}{3(k-1)} \left(\sum_{i=1}^{k} \frac{1}{n_i - 1} - \frac{1}{N-k} \right)$$

$$f_1 = k-1$$

$$f_2 = (k+1) / A^2$$

$$b = f_2/(1-A+2/f_2)$$

$$F = f_2 M / f_1 (b-M)$$

5.7.3.3 If $F > F_{\alpha; f_1, f_2}$ (where $F_{\alpha; f_1, f_2}$, is the $100\alpha\%$ point of the F distribution with f_1 and f_2 degrees of freedom), then the test casts doubt on the hypothesis of equal variances at the $100\alpha\%$ significance level. A program "BXTES*" has been written for the G. E. Mark II Time-Sharing Service to perform the above calculations - this program is listed in Appendix 2.

As pointed out in NRP-601, Box's test (which is a slight modification of Bartlett's test) can be severely affected by non-normality of the data. Positive kurtosis, for example, makes the true significance level of the test greater than the nominal figure.

5.7.4 Significance Level and Power of Tests for Homogeneity

In deciding on a significance level for a test of homogeneity of variances, two factors need to be considered. Firstly, the significance level is the probability of erroneously rejecting the hopothesis of equality when it is in fact true, and so from this point of view, we would like the significance level to be small. On the other hand, we would like the probability of rejecting the hypothesis when the variances are not equal (the power of the test) to be large. But for a given sample size, the smaller we make the significance level, the smaller the power of the test becomes, so we must compromise between the conflicting requirements of small significance level and large power.

5.0 Procedure (continued)

If we erroneously reject the hypothesis of equal variances when it is true, we may be led to using methods of analysis which are less satisfactory than those of Section 5.4.5. On the other hand, if we erroneously accept the hypothesis when it is false, and use the methods of Section 5.4.5, our lower tolerance limit calculations may be in serious error.

The power of tests for equality of variance is not great for moderate sample sizes. For example, if there are four cells with ten observations per cell, and the true variances are in the ratio 1:1:4:4, then the Box test with 5% significance level will correctly reject the hypothesis of equality for only 68% of all samples (i.e., with probability .68). Hence with probability .32 we would have a considerable error in our tolerance limit calculations.

To improve the power of the test for homogeneity of variances, a commonly used significance level in materials testing is 10%. This means that we will erroneously reject the hypothesis 10% of the time, but we cannot avoid all error with only a limited amount of information at our disposal.

5.7.5 Treatment of Unequal Error Variances

If the hypothesis of equality of variances is rejected, the first step to be taken is an investigation of the data, the testing procedures, the source of the material, etc., to see if there are any anomalies which might cast doubt on the validity of the data. There may be sound physical reasons for supposing that variances are unequal at different test conditions; but if the problem appears to be unequal variances in different lots, then the data should certainly be regarded as suspect.

5.7.5.1 Variance Stabilizing Transformations. If there appears to be no reason for doubting the validity of the test data, the next step is to check the possibility of transforming the data so as to achieve variance homogeneity. Such a transformation may be successful if there is, approximately, a simple relation between the cell means and cell standard deviations, i.e., $\sigma_i = h(\mu_i)$. In this case, transforming the data by the function g, where $g(x) = k\int \frac{dx}{h(x)}$, may produce approximate equality of variances. For example, if $\sigma_i \approx c\mu_i$ (i.e., standard deviation approximately proportional to mean) then the logarithms of the original data should have approximately equal variance. If $\sigma_i \approx c\sqrt{\mu_i}$, the square root is the appropriate transformation function. Cell means should be plotted agains cell standard deviations to see if there is a simple relation between the two, from which an appropriate variance-stabilizing transformation can be derived.

As discussed in NRP-601, if the coefficients of variation of the data (σ_i/μ_i) are small, then transformations like log x, $\sqrt{X}$ etc. should not have a significant effect on the normality of the data.

5.0 Procedure (continued)

5.7.5.2 Alternative Methods of Analysis. If after a transformation, a test for variance homogeneity indicates that the transformed data is homogeneous, then the methods of Section 5.4.5 can be applied. If a satisfactory variance-stabilizing transformation cannot be found, then other methods of analysis must be used. Some natural division of the data may be indicated, with variance equality within each division. For example, it may be possible to get a pooled estimate of error variance from data from different lots, but the same test condition. This estimate could then be used in deriving a lower tolerance limit at that test condition. Each case needs to be individually considered, and statistical assistance should be sought in dealing with these special analyses.

5.8 PROBLEMS OF INADEQUATE INFORMATION FROM TEST DATA

5.8.1 General

It may happen that, after test data for a material property are collected and analyzed, it is found that there is inadequate information contained in the data. Such a situation may occur if problems such as imbalance and heterogeneity of variances prevent the use of the powerful analytical methods of Section 5.4.5. Or it may occur simply because there were too few observations to achieve the precision of information required (for design, reliability assessment, etc.)

5.8.2 Unusable Lower Tolerance Limits

As an example, consider a case where n specimens are to be tested from each of I lots of material at a single test condition, and a lower 99/95 tolerance limit is to be found on the property in question. According to the method of Section 5.4.5.4, an approximate lower 99/95 tolerance limit is given by

$$\bar{Y}_{..} - k\, s_T$$

where $\bar{Y}_{..}$ is the grand mean, s_T is the estimated standard deviation of the entire population, and k is a tolerance factor whose size depends on n, I and R, the ratio of the lot-to-lot variance to the error variance. If I and/or n are small, and R is not very small, it can happen that k is quite large and, hence, that the lower 99/95 limit is very low. To illustrate, suppose I = 2, n = 10 and R = 1. Then k is about 6. If the true population mean and standard deviation are 40 and 2.5 respectively, this means that about 50% of the time (i.e., for about 50% of all samples), the lower 99/95 found will be 40 - 6 x 2.5 = 25 or less. (The true lower 99% point of the population is 40-2.326 x 2.5 = 34.18.) Such a number may be unusuable for design purposes. What is to be done to achieve a practicable design allowable?

5.0 Procedure (continued)

In the above example, we assumed that the true ratio of lot-to-lot variance to error variance was 1. When difficulties of the above kind occur in practice, the first step to be taken is a thorough investigation of the data, the testing procedures, the source of the material, etc., to determine if there are any anomalies which might cast doubt on the validity of the data; specifically, which might have resulted in an inflated estimate of lot-to-lot variance. It needs to be recognized, however, that very small lot sample sizes (I = 2, for example) contain little information about lot variability, and that a fairly large estimate of lot-to-lot variance may be legitimately obtained even if the true variance is relatively small. Furthermore, if the lot-to-lot variance is not small relative to the error variance, then the above example shows that the small number of lots tested will, in a large proportion of samples, inflict a heavy penalty on the design allowable. In these cases, the low design allowable is not due to the nature of the material, but is due to the fact that insufficient information about the material was obtained from the small number of lots tested.

5.8.3 Solutions to the Tolerance Limit Problem

The obvious answer to such problems is to test more lots, and that is what should be done if it is possible. But cost or time constraints may prevent the collection of more data, and hence some means must be found to get usable answers from the available data. Three methods are given below for estimating a lower 99/95 tolerance limit (design allowable) when the difficulty described in the example above occurs.

5.8.3.1 Estimated Upper Bound on Lot-To-Lot Variance. The first method requires that a conservative estimate (an upper bound) of the lot-to-lot variance be derived, using whatever relevant evidence is available. Such evidence may include published data on the material in question, experience with similar materials, information from vendors, etc. In no case should the assumed upper bound be less than the estimate of lot-to-lot variance calculated from the data. If, in the judgment of the responsible engineers, there is not sufficient evidence to support such a conservative estimate, then this method should not be used.

Let σ_B^2 be the assumed upper bound on lot-to-lot variance, let $\hat{\sigma}_\epsilon^2$ be the calculated estimate of error variance, having f = I (n-1) degrees of freedom, and let $\bar{Y}..$ be the grand mean. Then an approximate lower 99/95 tolerance limit is given by $\bar{Y}.. - 2.326\,\sigma_B - k'\hat{\sigma}_\epsilon$, where k' is found as follows: derive k from the computer program "TFAC**" with f = I (n-1) and

$$m = \frac{\sqrt{\sigma_B^2 + \hat{\sigma}_\epsilon^2} - \sigma_B}{\sqrt{\frac{1}{In}\,(n\sigma_B^2 + \hat{\sigma}_\epsilon^2)}} ,$$

and let $k' = k\left(\sqrt{\sigma_B^2 + \hat{\sigma}_\epsilon^2} - \sigma_B\right) \Big/ \hat{\sigma}_\epsilon$.

5.0 Procedure (continued)

(For designs other than that with I lots and n observations per lot (1 test condition), the values of f and m are calculated differently, and statistical assistance should be sought in applying this method.)

5.8.3.2 Interpolation Between Adjacent Test Conditions. It may happen that, when analyzing test data obtained at several test conditions, usable design allowables are derived for all test conditions except one intermediate condition. (For example, suppose test temperatures are -100, RT, 100, 200 and 300, and that only the design allowable at 200 was usuable.) If the usable values appear to lie on a smooth curve, and if there appears to be no physical reason for expecting a deviation from the curve at the test condition corresponding to the unusable value, then a design allowable can be obtained by reading the value from the curve at the test condition in question. This method cannot be used if the problem test condition is an extreme, i.e., lies at one end or the other of the range of conditions.

5.8.3.3 Use of the Lowest Lot Mean. If it can be established that adequate control can be exercised over material procurement, than a design allowable could be obtained by computing

$\bar{Y}_{[1].} - k\hat{\sigma}_{\epsilon}$, where $\bar{Y}_{[1].}$ is the lowest observed lot mean (calculated from m observations), $\hat{\sigma}_{\epsilon}^{2}$ is an estimate of error variance having f degrees of freedom, and k is the tolerance factor obtained from computer program "TFAC**" using the above-mentioned values of f and m.

The rationale of this method is that procurement control (through acceptance testing, etc.) will ensure that future lots will not be purchased if the lot mean falls below $\bar{Y}_{[1].}$, and hence the method cannot be used unless it is considered that such control can be successfully implemented.

6.0 APPLICABLILITY

6.1 The guidelines and methods discussed in this procedure are applicable in planning and analyzing materials properties tests.

6.2 Data Item R-106 will define and identify the methods whereby materials properties data will be used in reliability assessment of the NERVA engine and its parts.

7.0 RESPONSIBILITIES

7.1 The proper use of the methods described herein are the responsibility of each cognizant engineer using these techniques. These will include:

7.0 Responsibilities (continued)

7.1.1 Materials engineers in planning and analyzing materials properties tests.

7.1.2 Reliability engineers in assessment of reliability.

7.1.3 Design engineers in using materials data.

8.0 REFERENCES

(1) Hald, A., Statistical Theory with Engineering Applications, John Wiley (1952)

(2) Owen, D. B., Factors for One-sided Tolerance Limits and for Variables Sampling Plans, Monograph No. SCR-607, Sandia Corporation, Albuquerque (1963-1964)

(3) Scheffé, H., The Analysis of Variance, John Wiley (1958)

(4) Snedecor, G. W., and Cochran, W. G., Statistical Methods, Iowa State University Press (1969)

APPENDIX 1

COMPUTER PROGRAM "TFAC**"

TFAC** is a FORTRAN program which computes the lower 100P%/100γ% tolerance factor for given

(a) P = desired probability;
(b) γ = desired confidence level;
(c) m = sample size of the mean;
(d) f = degrees of freedom for the standard deviation.

(Note that m and f are not necessarily integers.) The user is also required to supply the number

Z_p satisfying $$P = \int_{-\infty}^{Z_p} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

(Z_p may be found in the usual tables for the cumulative normal distribution function - e.g., that in NRP-600, p. 148 ff.), and a starting constant for the program's binary search routine, which may be any number between 5 and 10.

An example of the input and output of the program (on the G.E. Mark II Time-Sharing Service) appears below:

```
                                  (P)  (γ)
DESIRED RELIABILITY & CONFIDENCE LEVEL? .99,.95
                                  (m)  (f)
SAMPLE SIZES OF MEAN & STD. DEVIATION.? 3.4,6.9
                                               (Zp)
INV. OF NORM. CUM. DIST. FUNC. (CORRES. TO REL.)? 2.326
STARTING CONSTANT FOR BINARY SEARCH ROUTINE (USE 5-10)?7

TOLERANCE FACTOR =     5.16517162
```

The program is listed below:

Appendix 1 (continued)

```
TFAC**                         09/16/70

C    PROGRAM TO COMPUTE TOLERANCE FACTOR, GIVEN DESIRED RELIABILITY,
C    CONFIDENCE LEVEL, SAMPLE SIZES FOR MEAN AND STANDARD DEVIATION,
C    & INVERSE OF NORMAL CUMULATIVE DISTRIBUTION FUNCTION CORRESPOND-
C    ING TO RELIABILITY.          (D. H. FEARN)
     REAL  K,K0,KP,N
     PRINT,"DESIRED RELIABILITY & CONFIDENCE LEVEL"; INPUT, P,GAMMA
     PRINT,"SAMPLE SIZES OF MEAN & STD. DEVIATIONS"; INPUT, N,F
     PRINT,"INV. OF NORM. CUM. DIST. FUNC. (CORRES. TO REL.)"
     INPUT, KP
     PRINT,"STARTING CONSTANT FOR BINARY SEARCH ROUTINE (USE 5-10)"
     INPUT, K0
```

Binary search routine to find tolerance factor

```
      NN=N
      FN=N
      D=K0
      SQFN=SQRT(FN)
      DELTA=KP*SQFN
      DO 3 I=1,20
      CALL H(F,K0*SQFN,DELTA,HH,DH)
      D=D/2.
      IF(HH-GAMMA)1,1,2
    1 K0=K0+D
      GO TO 3
    2 K0=K0-D
    3 K=K0
      PRINT 4, K
    4 FORMAT(/5X,"TOLERANCE FACTOR =",F15.8)
      STOP; END
```

Subroutine to compute non-central t dist.

```
      SUBROUTINE H(F,T,DELTA,HH,DH)
      REAL M(1000),MB,MM
      A=T/SQRT(F)
      B=F/(F+T*T)
      W=SQRT(2.*3.1415927)
      X=SQRT(B)
      NF=F+1./2.
      MB=A*X*DG(DELTA*X)*G(DELTA*A*X)
      AA=1.
      DO6K=1,NF
      IF(K-2)1,1,2
    1 M(1)=B*(DELTA*A*MB+A/W*DG(DELTA))
      M(2)=.5*B*(DELTA*A*M(1)+MB)
      GOTO6
    2 FK=K
      AA=1./((FK-2.)*AA)
      M(K)=(FK-1.)/FK*B*(AA*DELTA*A*M(K-1)+M(K-2))
     IF(M(K)-1.E-20)20,20,6
   20 M(K)=0.
    6 CONTINUE
      NNF=NF-2
      IF(NF-2*(NF/2))7,7,8
    7 MM=MB
      L=2
      Q=G(-DELTA)
      Y=W
      DH=F/T*M(NF)*W
      IF(NNF)9,9,10
    8 MM=0.
      L=1
      Q=G(-DELTA*X)+2.*T1(DELTA*X,A)
      Y=2.
      DH=2.*F/T*M(NF)
      IF(NNF)9,9,10
   10 DO11K=L,NNF,2
   11 MM=MM+M(K)
    9 HH=MM*Y+Q
      RETURN;END
```

Appendix 1 (continued)

Function to compute normal density

```
1000      FUNCTION DG(I)
1010      Z=SQRT(2.*3.1415927)
1020      DG=(1./Z)*EXP(-I*I/2.)
1030       RETURN$  END
```

Function to compute normal dist.

```
1100      FUNCTION G(I)
1110  IF(I)1,1,2
1120    2 G=1./2*(1.+ERRF(I/SQRT(2.)))
1130       GOTO3
1140    1 G=1./2.*ERRF(I/SQRT(2.))
1150      3 CONTINUE
1160      RETURN$END
```

Function to compute an integral by Simpson's rule

```
1200      FUNCTION TI(H,A)
1210      REAL Y(5)
1220      IF(H-12.)9,9,11
1230   11 TI=0.
1240      GOTO3
1250   9 PI=3.1415927
1260      Y(1)=(1./(2.*PI))*EXP(-H*H/2.)
1270      ERROR=0.00000001
1280      TI=0.
1290      LL=0
1300      J=7
1310      SE=0.
1320      DX=A/(2.**9)
1330   U=Y(1)
1340      L=1
1350   5 DO 1 I=1,L
1360   DO 15 II=1,4
1370      FII=II+LL
1380      X=DX*FII*(2.**J)
1390    IF(H*H/2.*(1.+X*X)-20)2,2,14
1400   2 Y(II+1)=(1./(2.*PI))*EXP(((-H*H)/2)*(1.+X*X))/(1.+X*X)
1410    GOTO15
1420   14 Y(II+1)=0.
1430   15 CONTINUE
1440      LL=LL+4
1450      E=(DX/45.)*(Y(1)-4.*Y(2)+6.*Y(3)-4.*Y(4)+Y(5))
1460       E=E*2.**J
1470      T=(2.*DX/45.0)*(7.*Y(1)+32.*Y(2)+12.*Y(3)+32.*Y(4)+7.*Y(5))
1480       T=T*2.**J
1490      Y(1)=Y(5)
1500      SE=SE+E
1510   1 TI=TI+T
1520      IF(ABS(SE)-ERROR)3,3,4
1530   4 J=J-1
1540      IF(J)3,3,6
1550   6 SE=0.
1560      IT=0.
1570      LL=0
1580      L=2*L
1590      Y(1)=U
1600      GO TO 5
1610   3 RETURN$  END
```

APPENDIX 2

COMPUTER PROGRAM "BXTES*"

An example of the input and output of the "BXTES*" program (on the G. E. Mark II Time Sharing Service) appears below:

```
NO. OF DATA SETS.......?  4

TYPE OF DATA SETS (VARIANCES=1, DATA=2)..?  2

SETS OF SAMPLE SIZE AND CORRESPONDING DATA
? 10, 35.1 34.3 35.0 3[illegible].4 33.6 34.7 33.0 34.2 34.6 37.1
VARIANCE = 0.10[illegible]1E+01

? 10, 30.1 30.6 302[illegible].2 29.9 30.3 3[illegible].1 29.3 29.2 31.7 28.6
VARIANCE = 0.84984E+00

? 10, 20.2 [illegible].1 22.6 21.1 20.2 21.3 21.4 19.7 20.2 20.9
VARIANCE = 0.[illegible]3539E+00

? 10, 19.2 [illegible].0 20.2 19.3 18.7 1[illegible].5 19.7 18.5 18.2 19.1
VARIANCE = 0.64849E+00



CRITICAL VALUE IS 0.20434E+00
WITH   3 AND 0.648000E+04 DEGREES OF FREEDOM
```

The program is listed below:

Appendix 2 (continued)

```
BXTLS*

C       PROGRAM TO COMPUTE RESULTS OF BOXES TEST OF VARIANCES
        DIMENSION V(25),N(25),X(25,100)
        PRINT,"NO. OF DATA SETS......"; INPUT, K
        PRINT,"TYPE OF DATA SETS (VARIANCES=1, DATA=2).."; INPUT, I
        GO TO (1,2),I
      1 PRINT 8; INPUT, (V(I),N(I),I=1,K)
        GO TO 5
      2 PRINT 9
        DO 4 I=1,K
        INPUT, L,(X(I,J),J=1,L)
        SX=0.
        SX2=0.
        DO 3 J=1,L
        SX=SX+X(I,J)
      3 SX2=SX2+X(I,J)**2
        V(I)=SQRT((SX2-SX**2/L)/(L-1))
        N(I)=L
      4 PRINT 10, V(I)
      5 DO 6 I=1,K
        N(I)=N(I)-1
        NSUM=NSUM+N(I)
        A=A+1./N(I)
        AVGVAR=AVGVAR+N(I)*V(I)
      6 PNLNV=PNLNV+N(I)*ALOG(V(I))
        AVGVAR=AVGVAR/NSUM
        A=(A-1./NSUM)/(3*(K+1))
        IF1=K-1
        F2=(K+1)/A**2
        B=F2/(1.-A+2./F2)
        XM=NSUM*ALOG(AVGVAR)-PNLNV
        CRIRAT=F2*XM/(IF1*(B-XM))
        PRINT 7, CRIRAT,IF1,F2
      7 FORMAT(//" CRITICAL VALUE IS",E12.5/" WITH",I4," AND",E13.6,
       &         " DEGREES OF FREEDOM")
      8 FORMAT(" SETS OF VARIANCES AND CORRESPONDING SAMPLE SIZE"/1X)
      9 FORMAT(" SETS OF SAMPLE SIZE AND CORRESPONDING DATA"/1X)
     10 FORMAT(" VARIANCE =", E12.5/1X)
        STOP; END
```

NERVA PROGRAM RELIABILITY PROCEDURE

NERVA PROGRAM RELIABILITY PROCEDURE	NUMBER: R001-G0P05	REVISION
	EFFECTIVE DATE: 12/9/71	CATEGORY II
FAILURE CONTROL SYSTEM IMPLEMENTATION	SUPERSEDES: NUMBER: DATE:	
	APPROVED BY: FRB CHAIRMAN J.A. Stewart PROGRAMS CONTROL [signature]	

1.0 **PURPOSE**

The purpose of the NERVA Program Failure Control System (FCS) is to assure that all failures are recognized and resolved so that design and reliability goals will be economically and effectively reached. To achieve this purpose the FCS is structured to identify unsatisfactory conditions that might result in failures and thus trigger action to eliminate the sources of these failures. The purpose of this procedure is to initiate the FCS on selected components during the CY'72 development program.

2.0 **GENERAL**

The Failure Control System, is established to provide a closed loop system for resolution of development test failures. The system is structured to provide adequate surveillance and audit of failure reporting to ensure objective analysis and corrective action. Figure 1 shows the interrelationships of the various elements of the FCS.

3.0 **SCOPE**

Departures classified as failures will be processed in accordance with this procedure. Departures classified as discrepancies will be processed by the Engineering Review Board (ERB) per Procedure NQP 15-1-21.

The Failure Control System (FCS) is applicable to all test article failures that occur during prototype Turbopump tests, TPA Bearing tests, LH_2 Pump Component Development tests, and prototype BCV valve actuator motor and gear tests.

Tests which are to be included in this system will be pre-declared in the monthly Failure Summary Report. Failures occurring due to any of the following conditions are specifically excluded from the FCS:

a) Failures of non-prototype equipment (i.e., off the shelf hardware being evaluated during the pre-design phase or workhorse hardware used as test support equipment).

b) Failures during screening of parts in which some items (but not all) are found to deviate from the required goals of the test.

c) Failures of the test article caused by a facility failure will be reported but will be closed out, with no further analysis, as soon as this cause is established. This closeout will be accomplished on the Failure Report. Failure of the test facility which does not cause failure of the test article will not be included in the failure control system, but will be documented and resolved utilizing the appropriate ANSC or NRTO procedures.

4.0 **DEFINITIONS**

4.1 **DEPARTURE**

A Departure is any discernable difference between the observations, events, and conditions that are preplanned and the actual results that are incurred; or any unpredicted or unexpected condition or behavior (including post-test teardown and analysis).

4.2 **FAILURE**

A failure is defined as a departure from 1) pre-declared predictions or 2) post test inspection criteria, including data analysis and disassembly.

4.3 DISCREPANCY

A discrepancy is any departure that is not classifed as a failure.

4.4 TEST

The operation or a coordinated series of operations of a part, subassembly or assembly conducted concurrently and/or in series, with the objective of determining facts about the article in relation to specified conditions. A test is used to (a) demonstrate the technical feasibility of a design, (b) determine its ability to meet existing performance requirements, (c) secure engineering data for use in design, or (d) establish the technical requirements for contract definition.

5.0 POLICY

5.1 FAILURE RESOLUTION BOARD

A Failure Resolution Board will be established with the Manager Engineering as Chairman with t e Assistant Manager as the alternate. The members will be the cognizant Project Manager, Manager Quality Assurance and Manager Reliability. The Manager, NERVA Engine Design shall assist the FRB Chairman as appropriate to ensure that engine level considerations are adequately reviewed. This Board will retain overall responsibility and authority for the establishment, implementation and operation of the Failure Control System. All decisions are the responsibility of the Chairman, but he will delegate responsibility for various elements of the system to existing organizational entities. The Reliability member will act as secretary for the FRB and presentations to the Board will be through the secretary. The FRB decisions that affect program cost, program schedule or items under formal change control will be coordinated through NERVA Rocket Operations Program Controls in accordance with the NERVA Operating Plan. The Chairman of the FRB will be responsible for providing full disclosure of all relevant information pertaining to proceedings of the FRB to the Government representative in a timely manner. The FRB Chairman shall, at his discretion, select the most appropriate means of communicating this information to the Government representative. The basic functions of the FRB are:

5.1.1 Resolve disagreements between FRB member organizations, departure classification, or Failure Analysis Plans.

5.1.2 Review the failure analysis and corrective action, approve Failure Analysis Reports, and approve final closeout o. corrective action when it has been shown that the basic cause of the failure has been defined and the corrective action is verified.

5.1.3 Monitor overall progress of the FCS by review of monthly Failure Summary Reports prepared by Reliability. Determine that the overall system is functioning properly and define actions as required to accomplish the purpose of the Failure Control System.

5.1.4 Approve monthly Failure Summary Reports transmitted to SNSO-C.

5.2 DOCUMENTATION OF TEST RESULTS

The Project Manager, through the cognizant engineer, will prepare a brief test prediction statement prior to each test to which the FCS applies and a Test Results Report after each test run and at the completion of post-test inspections performed by Quality Assurance and the cognizant engineer. The Test Results Reports shall be prepared within three days of test data availability or inspection completion. Any discernible difference between the predicted results and the actual results will be documented as a departure on an Inspection Report in accordance with NQP-15-1-20 and classified by the cognizant engineer as a Failure or Discrepancy. If a departure is noted or -classified after release of the initial Inspection Report, an amendment shall be issued with the new information. No special documentation or Test Results Report will be required of the cognizant engineer for departures recorded during fabrication and acceptance testing. Departures shall be classified as Failures or Discrepancies prior to disposition of the test article or affected test.

Failure Reports will be distributed by reliability within 5 days to each member of the FRB for their review and considered action.

Non-test related departures will be handled through existing EPB procedures as defined in NQP 15-1-21 Non-conforming Material Control and in the NERVA contract Preface and Support Documents.

5.3 CLASSIFICATION REVIEW

Reliability and Quality Engineering shall review departure classifications, Test Result Reports and summary analyses. If there is a disagreement by either Quality Engineering or Reliability with the cognizant engineer classification or analysis of any test result, an attempt will be made to resolve the disagreement. If this cannot be accomplished with the Project Manager, the disagreement shall be submitted within one week to the FRB chairman for resolution.

5.4 TESTING PRIOR TO FINAL ANALYSES

The Project Manager may continue testing if in his judgment no information relating to the failure investigation will be lost. Specific guidelines will be developed for each test as required.

5.5 FAILURE ANALYSIS

For each reported failure the Project Manager will be responsible for establishing, scheduling, preparing, and implementing a Failure Analysis Plan. Reliability will review and audit the progress of Failure Analysis Plans and investigations and will attempt to resolve any differences with the Project Manager or his designee. When differences cannot be resolved, the chairman of the FRB will be notified for resolution. A failure analysis including recommendations for corrective action shall be prepared by the Project Manager or his designee, reviewed by Reliability and Quality Assurance, and submitted to the FRB Chairman with Reliability and Quality Assurance comments, for approval.

5.6 CORRECTIVE ACTION AND FOLLOWUP

Corrective action, as defined by the Failure Analysis Plan, will be directed to the responsible departmental organization and implemented through existing management control operations and procedures. Status of all failure analyses and corrective action implementation will be maintained by Reliability and will be included in the monthly Failure Summary Report. The Project Manager will be responsible for accumulating and reviewing data subsequent to implementation of the corrective action, and based on this review, a recommendation will be made to the FRB to closeout a particular investigation. Failures will not be closed out until the failure mechanism is understood and the corrective action is verified. Reliability will review Test Results Reports to flag repetitive failure modes/mechanisms as well as to provide reliability assessment information.

5.7 DISCREPANCY ANALYSIS (TEST RESULTS AND HARDWARE)

For each departure reported as a Discrepancy, Quality Assurance will be responsible for evaluating the Discrepancy and determining the need for establishing and implementing an analysis. Whenever this analysis indicates an incorrect classification, or a related departure that may contribute to a test failure, a report shall be issued to the FRB and the need for analysis and corrective action determined.

5.8 DATA CENTER

A data center for test reports, operating time and cycle data, Failure Reports and corrective action will be maintained by Reliability. Quality Assurance will be responsible for maintaining a central point for all departures, discrepancy classifications, discrepancy analyses and corrective action results.

5.9 HARDWARE CONTROL

Failed hardware shall remain under control of FRB and protected to the extent necessary to preclude loss of evidence until such time as the failure analysis is completed.

6.0 IMPLEMENTATION RESPONSIBILITY AND INSTRUCTIONS

6.1 DEPARTURE REPORTING AND CLASSICATION

During development testing, the Project Manager will be responsible for preparing a pre-test prediction of the expected results of significant test parameters, (e.g., quantify the test objectives). These predictions shall be included in the individual test authorizing document (i.e., Test Request, Test Plan, or their supplements) shall be documented prior to the scheduled test and submitted to the FRB. All departures from these predictions or drawing requirements shall be documented on an Inspection Report (IR) by the Quality Assurance Department in accordance with NQP-15-1-20, and classified as a Discrepancy or Failure by the cognizant engineer. Each departure classification will be reviewed by the Reliability and Quality Assurance Departments. Agreement with the classification will be indicated by a sign-off on the IR. Disagreements on classification assignments will be resolved by the FRB chairman. When a departure is classified as a Failure, the IR document will be immediately closed out and processed as a Failure Report. The Quality Assurance Department will forward copies of all IR's to the Reliability Department within two working days of the classification decision.

6.2 FAILURE REPORTING

For those departures classified as Failures, Reliability will assign a Failure Report number and add the following information (if not already documented) to complete the Failure Report (FR):

a) Failure history of previous mode occurrences

b) Summary of previous mode occurrences

c) Operational usage at time of failure

d) Effect of failure

e) Failure effect category

f) Project Manager cognizant for failure analysis

6.3 FAILURE ANALYSIS

This plan shall outline disciplines necessary to support the analysis and check points where reassessments of initial steps shall be made. The plan as a minimum shall contain:

a) Documentation of existing evidence such as photographs of the area and hardware, records of the environment and conditions, a narrative description of the chain of events leading to the failure, and case histories of similar failures.

b) Disassembly findings such as photographs and measurements of step by step disassembly operations.

c) Evaluation of NERVA case histories of this type of failure on similar products.

d) Determination tests necessary that could provide data substantiating the parts conditions.

e) Analysis of evidence such as structural analysis of failed item under test loads and environments including the determination of the nature and validity of the mechanisms(s) of failure as well as the determination of the chain of occurrences and effects between the primary cause and all secondary failures.

f) Cause of Failure by postulating logic leading to Failure and substantiating with data.

g) Recommended corrective action by reviewing and selecting potential design, process, or procedural changes that could eliminate or significantly reduce the probability of occurrence of the failure mode.

6.4 FAILURE ANALYSIS REPORT

The format for the initial page of Failure Analysis Reports is shown in Figure 2. This report will be completed by the Project Manager or his designee and submitted to Reliability and Quality Assurance for review. Upon completion of the review cycle the report will be submitted to the FRB Chairman for formal approval. The failure analysis report shall contain the following information:

a) Failure History. This section will include a detailed chronological history of the processes leading to the Failure, a description of the Failure, and any applicable NERVA history of like Failures on similar parts.

b) Failure Analysis. This section will include the results of the analysis. The cause(s) of the failure shall be stated as well as the method and logic involved in determining the cause(s).

c) Corrective Action. This section will identify the actual steps taken or to be taken to remove the cause of failure. Examples of corrective action documentation are as follows:

1. For design corrections: The new drawing number or the dash number.

2. For performance corrections: The specification number revision letter.

3. For corrections to test parameters, environmental conditions, test procedures, test equipment inputs and other requirements relating to test or quality assurance provisions: The Test Plan or Test Request Supplement number.

4. For correction of supplier responsible actions: The issue number, revision letter or date on the corresponding supplier document, or purchase document.

5. For errors or conditions under customer responsibility: A statement to this effect shall be inserted.

d) Closeout plan. This section will show plans to obtain evidence that the corrective action implemented has indeed removed or significantly reduced the probability of the failure mode, and will define the action to be taken to assure that all parts fabricated to the failed configuration are reworked, retested, or removed from service.

6.5 FAILURE MODE CLOSEOUT

Upon completion of the failure analysis, the Project Manager will implement the plan to obtain evidence that the corrective action implemented has indeed removed or significantly reduced the probability of occurrence of the failure mode. Closeout requirements are:

a) Analysis and test data of the changed part or process that substantiate the cause determination and effectiveness of the change.

b) In the case of a rebuilt part, an analysis of non-failed components to determine if they have been overstressed.

c) For corrective action that affects other engine parts, an analysis of the effect of the redesigned part on engine performance.

The Project Manager or his designee will prepare a Failure Closeout Report for submittal to the FRB upon satisfaction of the above requirements. Upon agreement by the FRB that the corrective action has proven effective, the Failure will be closed out.

7.0 FAILURE SUMMARY REPORT

A Failure Summary Report will be prepared by the Reliability Department on a monthly basis to document the status of all failure analyses and corrective action implementation. These reports will be used to apprise the procuring activity of the types, severity, and relative frequency of hardware test failures; and of the status of remedial and preventive action being taken in activities to which the FCS applies. Each report shall contain the information listed below as required:

7.1 FAILURE REPORT LISTING

A section of each Failure Summary Report shall provide a columnar listing of principal data for each Failure Report (FR) generated during the report period. Data presented will include:

a) Failed component name, part, and serial number

b) Failure Report number and date

c) Test type and phase

d) Status of FR closure

e) Narrative failure description

f) Action responsibility (group or department)

g) Estimated failure analysis completion date

h) Failure type classification (criticality)

7.2 FAILURE INVESTIGATION STATUS

The status of each failure analysis will be presented.

7.3 FAILURE ANALYSIS RESULTS

A summary will be included of all analyses completed during the report period.

7.4 CORRECTIVE ACTION CLOSEOUT REPORT

A summary will be presented of each Failure Closeout Report approved by the FRB during the report period.

7.5 TABULATION OF CUMULATIVE FAILURE STATISTICS

A section of each Failure Summary Report shall provide a tabulation of cumulative statistics pertaining to failure modes reported to the FRB under the contract from its initiation through the data compilation cut-off date for that report period.

Each failure mode will be identified as to investigation and analysis status. Failure modes that have been closed out will be so indicated, but will remain in the failure mode tabulation.

7.6 APPLICABLE TESTS PLANNED

A listing and description of the tests to which the FCS applies planned for the subsequent report period will be included. This section will include a description of the criteria to be used for each test to determine failures.

8.0 DOCUMENT DISTRIBUTION

In addition to any standard distribution, FRB documentation will be distributed as follows:

8.1 Documents containing pre-test predictions will be distributed to Failure Resolution Board members, Reliability Document Center, and Quality Assurance Document Center.

8.2 Test Results Reports will be distributed to Failure Resolution Board members, Reliability Document Center, and Quality Assurance Document Center.

8.3 Inspection Reports bearing failure classification will be distributed to the cognizant project engineer, FRB members, Reliability Document Center, and Quality Assurance Document Center.

8.4 FAILURE REPORTS will be distributed to the cognizant project engineer, quality assurance and FRB members.

8.5 FAILURE ANALYSIS PLANS will be distributed to FRB members, Reliability Document Center, and Quality Assurance Document Center.

8.6 FAILURE ANALYSIS REPORTS will be distributed to FRB members, Reliability Document Center, and Quality Assurance Document Center.

8.7 FAILURE CLOSEOUT REPORTS will be distributed to FRB members and to the Reliability & Quality Assurance Document Centers.

8.8 FAILURE SUMMARY REPORTS will be distributed to FRB members, Engineering Department Manager, and procuring activity (SNSO).

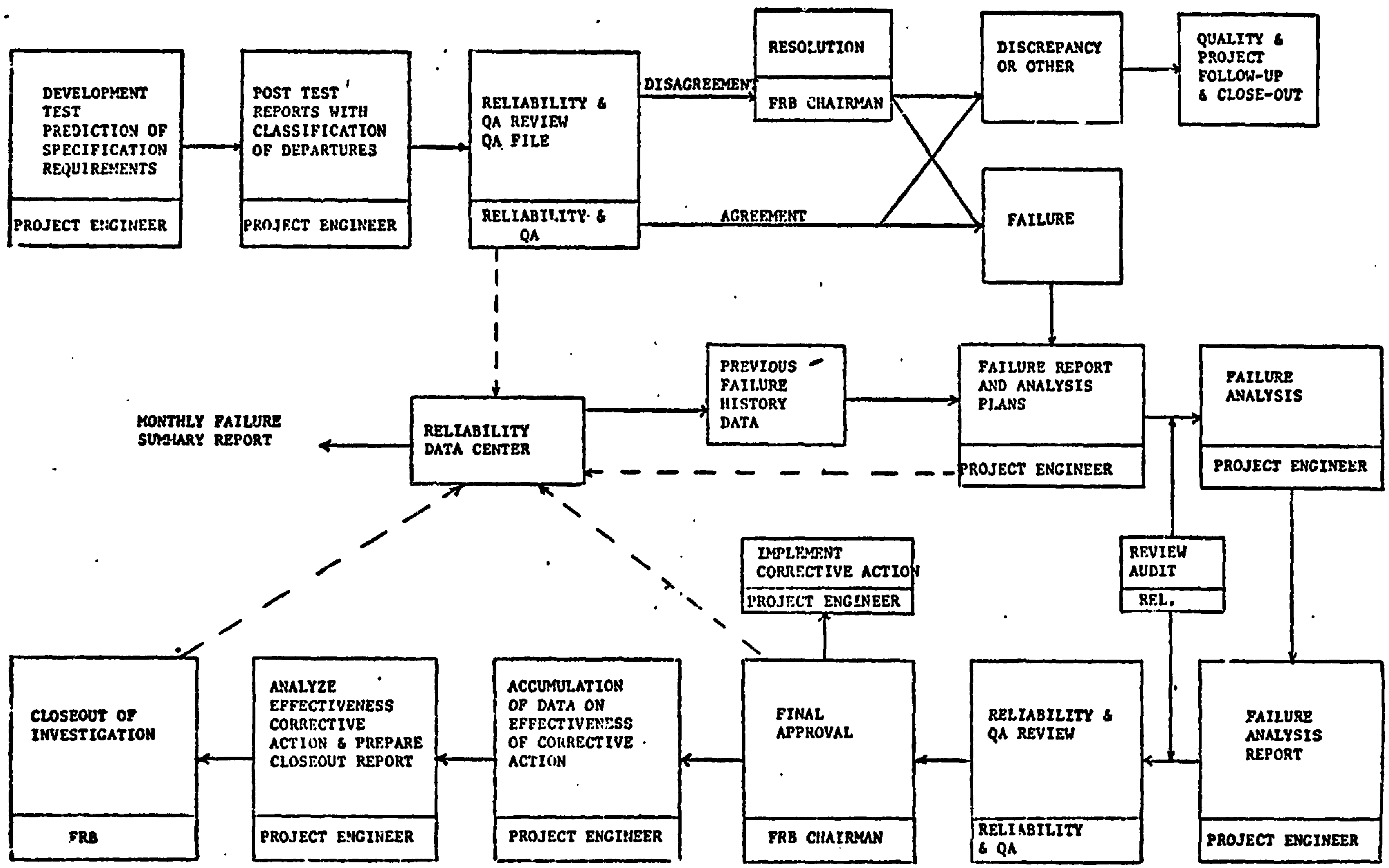

DEVELOPMENT TEST PREDICTION OF SPECIFICATION REQUIREMENTS
PROJECT ENGINEER
POST TEST REPORTS WITH CLASSIFICATION OF DEPARTURES
PROJECT ENGINEER
RELIABILITY & QA REVIEW QA FILE
RELIABILITY & QA
DISAGREEMENT
AGREEMENT
RESOLUTION
FRB CHAIRMAN
DISCREPANCY OR OTHER
QUALITY & PROJECT FOLLOW-UP & CLOSE-OUT
FAILURE
MONTHLY FAILURE SUMMARY REPORT
RELIABILITY DATA CENTER
PREVIOUS FAILURE HISTORY DATA
FAILURE REPORT AND ANALYSIS PLANS
PROJECT ENGINEER
FAILURE ANALYSIS
PROJECT ENGINEER
REVIEW AUDIT
REL.
IMPLEMENT CORRECTIVE ACTION
PROJECT ENGINEER
CLOSEOUT OF INVESTIGATION
FRB
ANALYZE EFFECTIVENESS CORRECTIVE ACTION & PREPARE CLOSEOUT REPORT
PROJECT ENGINEER
ACCUMULATION OF DATA ON EFFECTIVENESS OF CORRECTIVE ACTION
PROJECT ENGINEER
FINAL APPROVAL
FRB CHAIRMAN
RELIABILITY & QA REVIEW
RELIABILITY & QA
FAILURE ANALYSIS REPORT
PROJECT ENGINEER

FIGURE 1.

FAILURE ANALYSIS - Aerojet Nuclear Systems Company	ANALYSIS DATE	FAILURE REPORT NO.	FAILURE ANALYSIS REPORT NO.

PART NO.	SERIAL NO.	COMPONENT	MANUFACTURER

COGNIZANT ENGINEER	PROJECT MANAGER	PRB APPROVAL	
NAME	NAME	NAME	
SIGNATURE	SIGNATURE	SIGNATURE	

FIGURE 2

NERVA PROGRAM RELIABILITY PROCEDURE	NUMBER: R101-NRP-506	REVISION III
	EFFECTIVE DATE: 6 July 1970	CATEGORY
IDENTIFICATION AND CONTROL OF TREND CHARACTERISTICS	SUPERSEDES: NUMBER: DATE:	
	APPROVED BY: W. M. Bryan	

1.0 PURPOSE

1.1 The purpose of the NERVA trend data program is to identify and provide for the monitoring of selected critical characteristics of the NERVA engine system, subsystems and components which are related to wear and deterioration and which could indicate an incipient failure prior to completion of the test or mission. Such information could be used to avoid or prematurely terminate a [illegible], prepare for a possible alternative mode of operation, schedule corrective maintenance, or provide data for redesign to reduce the rate of wear or deterioration.

2.0 APPLICABLE DOCUMENTS

2.1 Data Item R-101, NERVA Reliability Program Plan, including supplement identified as RN-S-0539, Preliminary Implementation Plan for NERVA Trend Data Program, dated 15 October 1969.

3.0 POLICY

3.1 Trend characteristics (TC's) will be identified during preliminary design with the objectives of:

3.1.1. Developing a detection, monitoring and control system to prevent component, subsystem and engine tests from being performed that have a high probability of failure due to trend characteristic degradation stemming from wear or deterioration.

3.1.2. Developing a detection, monitoring and control system to provide timely warning that there is a high probability of a wear or deterioration related malfunction occurring prior to the end of the scheduled duration of a component or subsystem test, engine ground test, or engine flight operation.

3.1.3. Obtaining deterioration and wear out trend data for developing a flight maintenance diagnostic system to provide optimum scheduling of corrective maintenance between flight missions.

3.1.4. Providing for the recording of trend data, during the development test program, to be used for use by design and systems engineering for improving the performance, reliability, maintainability and safety of the NERVA engine system.

3.2 Depending on objectives, environments and requirements, differing trend characteristic parameters may be identified for component or subsystem tests, ground engine tests and engine flight tests.

3.3 Since trend characteristics represent only a portion of the total characteristics of an engine and are normally only measured during or after tests, their monitoring is not a substitute for the normal process control or quality assurance data monitoring. However, in some instances, the need to obtain significant trend data may require measurements or tests early in the fabrication process.

3.4 Although a trend characteristic may be monitored as part of the malfunction detection system, the malfunction detection system is also concerned with sensing (a) critical parameter trends that might also indicate incipient failure but which are not related to wear or deterioration, (b) critical parameters that are essentially binary in nature and would not be expected to give advanced warning of failure.

4.0 DEFINITIONS

4.1 Trend characteristics are variable parameters related to wear or deterioration that are indicators of critical component or engine failure modes and which can be measured either prior to, during, or after tests or flight operations. They must be variable in the sense that they would be expected to show a deteriorating change in average value, and/or dispersion, as a function of number of tests, test duration, or total accumulated time. As indicators, they must normally be expected to show the effects of wear or deterioration at a rate which can be monitored and used to avoid initiating a test, plan for its premature termination, or schedule future corrective maintenance. They must be related to critical failure modes that could cause significant performance or safety degradation during tests of components, subsystems and engines and which are credible in the sense that engineering judgment and past experience indicates a reasonable probability of occurrence. As wear and deterioration related parameters, they would involve one or more of the following manifestations:

4.1.1. Surface physical wear due to sliding or rolling contact.

4.1.2. Surface or internal chemical deterioration due to heat, radiation, etc.

4.1.3. Electronic property degradation due to heat, radiation, etc.

4.1.4. Surface corrosion due to contact with fluids, gases, lubricants, etc.

4.1.5. Internal mechanical property degradation due to creep and relaxation, cold flow, etc

4.1.6. Mechanical, chemical or electronic degradation due to manufacturing processes, assembly procedures, and test procedures.

Note: There may be many variable parameters whose trends are monitored for engineering, diagnostic, or malfunction detection purposes that will not be considered as part of the trend data program. Only parameters expected to reflect wear or deterioration effects are identified as trend characteristics.

5.0 PROCEDURE

5.1 Preliminary identification of trend characteristics will be made during preparation of component FMA's and during the preliminary design of selected concepts.

5.2 During the identification phase, trend characteristics will be selected in accordance with the definitions of Paragraph 4.0. The objectives of monitoring trend characteristics will vary depending on the period of surveillance as follows:

5.2.1. During the component or subsystem tests, trend characteristics will be monitored with the primary objective of preventing failures and avoiding damage to expensive test facilities. A secondary objective is gathering trend data to establish deterioration rates and limits for the same characteristics when measured on an engine system test or to establish that no appreciable degredation will occur prior to the end of service life and therefore no engine test monitoring will be required.

5.2.2. For engine ground tests, trend characteristics should be selected with the additional objective of providing information for optimum scheduling of corrective maintenance for ground tests and developing flight operation maintenance requirements. Care must be taken, therefore, in selecting parameters that will be uniquely related to a single replaceable component to avoid ambiguity in fault isolation.

5.2.3. For engine flight tests, the objectives are the same as 5.2.1. and 5.2.2., however, the limitations on the number of data channels and the inability to measure non-instrumented trend characteristics during flight must be recognized.

5.3 The designated preliminary trend characteristic, the suggested method of measurement the failure mode to which it is related, recommended period of surveillance (fabrication test and inspection, component or subsystem tests, engine ground test, engine flight test, etc.) and time of measurement (before/between or during tests) will be documented on the Trend Characteristics Identification Sheet (See Figure 1). Guidelines for the Trend Characteristics Identification are given in Figure 2.

5.4 Copies of the Trend Characteristic Identification Sheet will be retained by the originator, with the original kept in a master file from which official listings are prepared. Copies of TC's will be circulated for review by the affected disciplines such as safety, controls and instrumentation, quality engineering, test planning, and data collection.

5.5 A review for safety considerations will verify the completeness of the list and the criticality of the characteristics identified.

5.6 An evaluation of the capability of available instrumentation to monitor identified TC's during the designated tests will be made to determine the need for development or improvement of appropriate measurement systems. Recommendations regarding the suitability of present instrumentation or the need for additional development will be made on the TC Identification Sheet and returned to the coordination organization.

5.7 A review will be made of the capability of available inspection and measurement devices to measure trend characteristics before or after ground tests of components, subsystems or engines. Recommendation regarding the suitability of present techniques or the need for additional development will be made on the TC Identification Sheet and returned to the coordinating organization. This same review will also include evaluation of the feasibility of monitoring selected TC's during early fabrication phase inspection and tests.

5.8 An evaluation will be made of the feasibility of measuring or monitoring the TC's before, during or after tests. Recording and display channel availability and the general impact on test operations of the monitoring, recording and control activities relative to the selected TC's will be considered.

5.9 Following these reviews and evaluations, the preliminary Trend Characteristic Identification Sheets will be revised and an updated summary listing will be prepared and included in the Data Item R-202's submitted for engine PDR and component PDR.

5.10 A preliminary listing will be made at engine PDR, followed by an updated summary presentation at component PDR justifying the selected trend characteristics.

5.11 Subsequent to engine and component PDR's, the TC lists for components, subsystems, engine ground tests, and engine flight tests will be continually updated as additional information and experience is obtained. Revision to the TC lists will be included in subsequent R-202 reports at significant program milestones.

5.12 Trend characteristics will be incorporated and identified as requiring monitoring or measurement in component and subsystem specifications. TC's to be measured in engine ground tests will be included in the ground test MRL's if they involve instrumentation and in the specification if they involve other types of measurement before or between tests. All TC's to be measured in flight will be listed in flight MRL's.

5.13 The lists of identified TC's will also be used to initiate preliminary planning of a trend data information system that will store results of trend data measurements for engineering design purposes.

6.0 APPLICABILITY

6.1 A TC Identification Sheet shall be prepared, as specified in Section 5.0, for each selected TC for the components and subsystems listed in the NERVA Engine Specification Tree.

7.0 RESPONSIBILITY

7.1 Component Design Engineering shall be responsible for:

7.1.1. Identifying preliminary trend characteristics on TC Identification Sheets in coordination with Reliability Engineering and Engine Design.

7.1.2. Revising preliminary TC Identification Sheets in concordance with recommendation made by Controls and Instrumentation, Quality Engineering, Test Operation Planning and Safety Analysis.

7.1.3. Preparing summary presentations for component PDR in coordination with Reliability Engineering.

7.1.4. Incorporating provisions in component designs for instrumentation sensors for approved TC's.

7.1.5. Incorporating and identifying approved TC's in the component specifications.

7.2 System Design shall be responsible for accomplishing tasks similar to those stated in 7.1 in reference to the engine. In addition, Engine Design will add or identify trend characteristics in ground test and flight MRL's.

7.3 Reliability Engineering shall be responsible for the overall coordination of the trend data program and shall have final approval of the selected characteristics. In addition to assisting Component Design Engineering and Engine Design in the TC selection, revision and justification activities, Reliability Engineering shall:

7.3.1. Maintain the official TC summary listing of and be responsible for distribution and follow-up activities for TC review by Controls and Instrumentation, Quality Engineering, Test Operations Planning and Safety Analysis.

7.3.2. Be responsible for including the updated summary listing of TC's in Data Item R202's submitted for engine and component PDR and at subsequent program milestones.

7.3.3. Forward lists of preliminary and updated summary listings of TC's to Data and Configuration Control for use in information storage and retrieval system planning.

7.4 Controls and Instrumentation shall be responsible for:

7.4.1. Evaluating capability of available instrumentation to monitor the identified preliminary TC's during component tests, engine ground tests, and during flight operations.

7.4.2. Making recommendations relative to the need for additional instrumentation development, where needed, to meet TC monitoring requirements.

7.5 Quality Engineering shall be responsible for:

7.5.1. Evaluating the capability of available inspection and measuring devices to measure trend characteristics before or after ground tests of components or engines.

7.5.2. Evaluating the feasibility of monitoring selected TC's during early fabrication phase inspections and tests.

7.6 Test Operations Planning shall be responsible for:

7.6.1. Evaluating the feasibility of measuring or monitoring the TC's before, during, or after tests.

7.6.2. Making recommendations regarding channel availability and general impact upon test operations of the proposed monitoring and control activity.

7.7 Safety Analysis shall be responsible for reviewing the preliminary TC Identification Sheets to assure that the trend characteristics identified are related to critical failure modes and that all wear or deterioration caused failure modes that can result in significant performance or safety degradation are represented.

7.8 Additional responsibilities for establishing sampling frequency, trend limits, action criteria for limit exceedence, and the relationship with malfunction detection and fault isolation systems will be covered in Procedures R 101-NRP-507 and R101-NRP-508.

NERVA PROGRAM

TREND PARAMETER IDENTIFICATION SHEET

BASIC DATA

1. Trend Parameter Identification Number TP - ____________

2. Parameter Name ____________

3. Suggested Method of Measurement ____________

4. Related Trend Characteristics (Failure Modes) ____________

____

5. Recommended Period of Surveillance:

		Applicable Reference Document		
		Drawing No.	Spec. No.	MRL No.
Fabrication Inspection or Test	☐	______	______	
Component Test	☐	______	______	
Subsystem Test	☐	______	______	
Engine Ground Test	☐	______	______	______
Engine Flight Test	☐	______	______	______

Note:

6. Time of Measurement:

	Pretest	Posttest
No Disassembly:	☐	☐
Disassembly Required:	☐	☐
During Tests:	☐	

7. Type of Degradation Sensitivity: Operation Time ☐ Cycles ☐ Both ☐

SUPPLEMENTAL INFORMATION

8. Instrumentation or Measurement Analysis ____________

____

____

____

____

9. Test Operations Analysis ____________

____

____

____

____

APPROVED

____________ ____________

Originator Trend Data Program Coordinator

Figure 1 (cont.)

TREND PARAMETER IDENTIFICATION

CONTINUATION SHEET

Trend Parameter Identification Number______________________________

Parameter Name______________________________

____

Method of Measurement or Calculation______________________________

____

____

____

____

____

____

____

____

Related Trend Characteristics

Originating Wear/Deterioration Mechanisms	Resulting Failure Modes
______________	______________
______________	______________
______________	______________
______________	______________
______________	______________
______________	______________
______________	______________
______________	______________
______________	______________
______________	______________
______________	______________

Recommended Surveillance Program______________________________

____

The Purposes for Monitoring Trend Characteristics

The purpose of the NERVA trend data program is to identify and monitor critical wear or deterioration-related characteristics of the NERVA engine, subsystems or components which might indicate an incipient failure prior to the completion of the test or mission. Such indications of wear or degradation can be used to:

a. Provide data for component redesign to reduce the rate of wear or deterioration.

b. Avoid initiating a component subsystem or engine ground test that could result in a malfunction.

c. Give warning of the need to prematurely terminate a component, subsystem, or engine ground test.

d. Provide advance warning of an incipient failure during flight that may require crew action such as switching to an emergency mode of operation.

e. Provide advance warning of the need to schedule corrective maintenance prior to the next ground test or flight mission.

How a Trend Characteristic is Defined

A trend characteristic must:

a. Be a continuous variable (as opposed to discrete or binary).

b. Be related to credible failure mode that could cause significant performance or safety degradation.

c. Be measurable either prior to, during, or between tests.

d. Be expected to show deterioration trends in average or dispersion as a function of numbers of tests, test duration, or total accumulated time.

e. Be uniquely related to a given part or component to avoid ambiguity in fault isolation.

f. Be a result of such wear or deterioration manifestations as:

(1) Surface physical wear due to sliding or rolling contact.

(2) Surface or internal chemical deterioration due to heat, radiation, etc.

(3) Electronic property degradation due to heat, radiation.

(4) Surface corrosion due to contact with fluids, gases, lubricants, etc.

(5) Internal mechanical property degradation due to creep, relaxation, cold flow, etc.

(6) Mechanical, chemical or electronic degradation due to manufacturing processes, assembly procedures, and test procedures.

Note: There may be many variable parameters whose trends are monitored for diagnostic or malfunction detection purposes that are not part of the trend data program. Only parameters expected to reflect wear or deterioration effects should be identified as trend characteristics.

Guide to Filling out the Trend Characteristic Identification Sheet

a. Review the Failure Modes Analyses (FMA's) and Failure Mode Effects and Criticality Analyses (FMECA's) applicable to the component, subsystem or system being analyzed.

b. Initiate a Trend Characteristic Identification Sheet for each failure mode, or group of closely related failure modes, that involve wear or deterioration with operating time or cycles of use.

c. Enter under "Parameter Name" the characteristic that can best be monitored to detect the wear or deterioration manifestation.

d. Enter the suggested method or methods of measurement and the failure mode or modes that can be monitored by the identified parameter.

e. Indicate the period or periods of surveillance being considered. (Note: It may be necessary to initiate different TC Identification Sheets for different periods of surveillance.)

f. Indicate the applicable reference document where the trend characteristic can or should be identified for the selected period of surveillance.

g. Indicate under "Time of Measurement" whether the parameter would normally be measured before or between tests (with or without disassembly) or during tests.

h. Indicate under "Type of Degradation Sensitivity" whether the TC is expected to degrade as a function of time, cycles, or both.

NERVA PROGRAM RELIABILITY PROCEDURE	NUMBER: R101-NRP-507	REVISION
	EFFECTIVE DATE: 21 August 1970	CATEGORY III
DEVELOPMENT AND IMPLEMENTATION OF TREND CHARACTERISTIC MONITORING SYSTEMS	SUPERSEDES: NUMBER: DATE:	
	APPROVED BY: J. W. Pemrick	

1.0 PURPOSE

1.1 The purpose of this procedure is to establish the requirements for the development and operation of a comprehensive system for the monitoring of wear and deterioration-related characteristics that can affect system reliability. Provision is made for the initial recording of trend data during and after component development tests with either real-time or post test monitoring. The responsibilities for establishing trend action limits, sampling and analysis frequency, and other trend projection hardware and software details for both engine ground tests and the flight operational system are also covered.

2.0 APPLICABLE DOCUMENTS

2.1 Data Item R101, NERVA Reliability Program Plan

2.2 R101-NRP-506, Identification and Control of Trend Characteristics

2.3 R101-NRP-508, Analysis and Verification of Trend Characteristics

3.0 POLICY

3.1 The NERVA engine shall have the capability to calculate the probability of mission success at any time during a mission. Since trend data provides one of the inputs to such reliability calculations, provision shall be made for real time monitoring and analysis of trend parameters during flight operations. Capability shall be incorporated for predicting and displaying time remaining prior to component wearout or deterioration in excess of trend action limits.

3.2 As part of the reliability program, wear and deterioration related trend characteristics shall be monitored during component development and qualification tests at the subsystem and engine levels with the following objectives:

3.2.1 Prevent malfunctions by terminating a test in progress or identifying required corrective maintenance prior to test initiation.

3.2.2 Obtain data to identify components that must be redesigned because of excessive rates of wear or deterioration.

3.2.3 Obtain data to establish trend characteristic monitoring and maintainability requirements for engine flight operations.

3.3 Details of trend characteristic monitoring such as sampling frequency, trend action limits, trend projection techniques, and action to be taken when operation in excess of action limits is predicted, shall be specified in component, subsystem and engine test plans. Format details for display plotting during tests and for post test trend graphing shall also be identified. Where some or all of these functions are performed by the instrumentation and controls subsystem in ground or flight tests they shall also be defined in the specifications for that subsystem.

3.4 Instances of actual values exceeding trend action limits during a test will be classified as a departure from a requirement and will be reported to Engineering or Material Review Boards for classification as a discrepancy or as a failure. A similar reporting policy applies to instances where the test is terminated because the trend projection technique indicated the action limit would be exceeded prior to normal test duration, even though no actual limit violation occurred.

3.5 The proper interpretation of results of trend characteristic monitoring will usually require knowledge of cumulative test time or cycles of use. Provisions for acquiring of such time or cycle data shall be included in component or subsystem specification.

4.0 DEFINITIONS

4.1 Trend Characteristics are variable parameters that provide a measure of the effects of wear or deterioration. They are one of the inputs that may be used in estimating system reliability.

4.2 Trend Characteristic Monitoring is the process of obtaining successive measurements of trend characteristics for a given serial numbered component, subassembly or engine; plotting or analyzing the data; projecting a trend; and predicting if and when an established trend action limit will be exceeded.

4.3 Data Compression Technique is the method used to avoid the recording and processing of non-significant data.

5.0 PROCEDURE

5.1 For each component, subsystem, or engine test or test series where trend characteristics are to be monitored, the following trend characteristic monitoring requirements (illustrated in Figure 1) shall be determined and specified:

5.1.1 The Type of Trend Comparison, which distinguishes the test-to-test monitoring of the same article from the within-test monitoring at intervals during a test is determined by whether the parameter of interest can be measured only at discrete times or is continuously monitored.

5.1.2 The Sampling Interval, which describes the number of seconds, or cycles, between measurements, applies primarily to within-test monitoring. The choice of interval is influenced by instrumentation or measurement capability, expected rapidity of trend deterioration, and type of trend data processing or trend projection technique to be employed.

5.1.3 The Trend Parameter Range, which describes the maximum total range that is of interest to the designer. It shall provide for a margin of measurement beyond the specification or action limit range in order to assist in failure analysis and limit re-evaluation activities.

5.1.4 The Trend Action Limits, which establish the point at which action shall be taken to terminate the test or to initiate other appropriate measures. During preliminary design, trend action limits may be tentatively established by engineering estimate or systems analysis subject to later validation or revision by analysis or test. Upon validation, trend action limits shall be incorporated in the reliability requirements section of the specification. Where the limits change during the duration of the tests, a time/limit envelope shall be specified or provision made to sample only at a specific time during an operational phase or when some related parameter reaches a specific reference level.

5.1.5 The Test Duration which is the planned duration of the test or test series in time or cycles.

5.1.6 The Estimated Measurement Accuracy Tolerance which is the band about a given reduced data point which has an estimated 99.73% probability of containing the true value. This tolerance band should reflect an estimate of both the bias and imprecision.

5.1.7 The Trend Projection Technique, which is the method used to establish the current trend of the monitored parameter and extrapolate this trend into the future. (This could take the form of fitting a straight line or polynomial curve to the first group of points, establishing a measurement accuracy tolerance band about this projected trend, and recalculating the projected trend whenever a specified number of points in succession fall outside of the tolerance band.)

5.1.8 The Monitoring, Plotting, Projection and Limit Violation Warning system, which describes the combination of hardware, software, and personnel used to accomplish the trend characteristic monitoring activities. (For tests where failure due to exceeding the trend limit could be hazardous or costly, an extensive system involving real time data reduction, display of points and calculated trend projection on a cathode ray tube, plus visual and audible warning of limit violation might be required. A less costly system might substitute oscillographs for cathode ray tube display, but still have real time calculation of trend projection and warning of limit violation. Where exceeding the trend action limit has less serious consequences, or where comparisons are only made from test-to-test, other manual plotting and trend projection techniques may be adequate. Where greater accuracy is required, and rapidity of trend evaluation is not critical, a system of computer plotting and trend projection based on reduced and verified data may be the best choice. Limitations of data storage capacity, such as in the engine flight system, may require data compression techniques to reduce stored data to the minimum needed to accomplish the essential trend characteristic monitoring objectives).

5.2 Instructions for forwarding data for storage or further analysis processing and summation shall be included when specifying trend monitoring requirements.

5.3 Proposed trend characteristic monitoring requirements shall be reviewed by affected disciplines such as test operations, controls and instrumentation, engine design, and computer services. The impact on test facilities, equipment, personnel and operations shall be evaluated and appropriate action initiated.

5.4 The requirement to monitor approved trend characteristics shall be incorporated in appropriate specifications, and the implementing details shall be reflected in test and inspection planning documents. Those requirements affecting test facilities equipment shall be incorporated in planning data for facilities and special test equipment. Design for both ground and flight instrumentation and control subsystems will incorporate necessary features to accomplish the required trend data monitoring activities.

5.5 A summary of planned trend data monitoring activities shall be presented at demonstration PDR. Supplementing details shall be reported at subsequent program milestones.

5.6 The monitoring system shall be implemented during development prequalification and qualification testing of components, subsystems, and engines. In addition to on-site monitoring, a centralized data collection, monitoring and summary analysis system shall be operated by the Reliability Data Center as described in R101-NRP-508. Results shall be continuously reviewed to determine the need for expansion or reduction of trend data monitoring and to determine the flight monitoring system requirements.

6.0 APPLICABILITY

6.1 Trend data monitoring requirements shall be established for each approved trend characteristic. Different monitoring system requirements may be specified for different test types, conditions, or locations.

7.0 RESPONSIBILITY

7.1 Reliability shall have overall responsibility for the trend data program and will initiate the establishment of trend characteristic monitoring requirements for each approved trend characteristic. Monitoring system detailed requirements for components, engine ground tests, and flight operations will be developed in collaboration with Component Design Engineering, Engine Systems Design, and Controls and Instrumentation. Reliability will be further responsible for:

7.1.1 Securing approval of the proposed monitoring techniques from the affected test organization.

7.1.2 Preparing a summary of the approved monitoring system for presentation at demonstration PDR.

7.1.3 Collecting, analyzing and summarizing trend data as part of the Reliability Data Center operation.

7.2 Component Design Engineering shall be responsible for:

7.2.1 Initiate development and monitoring system detailed requirements in conjunction with reliability.

7.2.2 Incorporating trend characteristic monitoring requirements into appropriate specifications, drawings, and test requirement documents.

7.2.3 Reviewing and analyzing results of trend data monitoring.

7.3 Engine System Design Shall be Responsible For:

7.3.1 Reviewing proposed trend monitoring requirements for engine ground test and flight operations in terms of instrumentation and controls capability and related requirements of malfunction detection, fault isolation, and operating controls.

7.3.2 Coordinating the preparation and updating of measurement requirement lists for ground tests and the flight engine to identify measurements required in the monitoring of trend data characteristics.

7.4 Test Operations Shall be Responsible For:

7.4.1 Reviewing proposed trend characteristic monitoring requirements for feasibility in terms of test facilities, equipment, personnel and operating procedures.

7.4.2 Incorporating approved trend characteristic monitoring requirements into planning data for facilities and special test equipment.

7.4.3 Incorporating trend characteristic monitoring requirements into detailed test planning for components, subsystems, and engines.

7.4.4 Operating the trend characteristic monitoring system during tests, taking appropriate action when limit violation is predicted, and forwarding data to the Reliability Data Center for analysis and summarization upon test completion.

7.5 Computing Services Shall be Responsible For:

7.5.1 Reviewing proposed monitoring system requirements for component, subsystem and engine tests in terms of computer processing capability.

7.5.2 Coordinating development of the trend monitoring hardware and software system with Test Operations and Controls and Instrumentation.

7.6 Controls and Instrumentation Shall be Responsible For:

7.6.1 Reviewing proposed engine monitoring system requirements for feasibility in terms of instrumentation and control subsystem capability.

7.6.2 Incorporating the necessary features in the instrumentation and control system design to accomplish the detection, processing, and analysis of trend data in real time during flight operations.

7.6.3 Integrating trend characteristic monitoring requirements with the related instrumentation, analysis, and display requirements for malfunction warning, fault isolation, and continuous mission success probability evaluation.

7.6.4 Coordinating the design, development and procurement of ground test trend monitoring software and hardware with Test Operations, Computing Services, and Reliability.

7.7 Quality Assurance Shall be Responsible For:

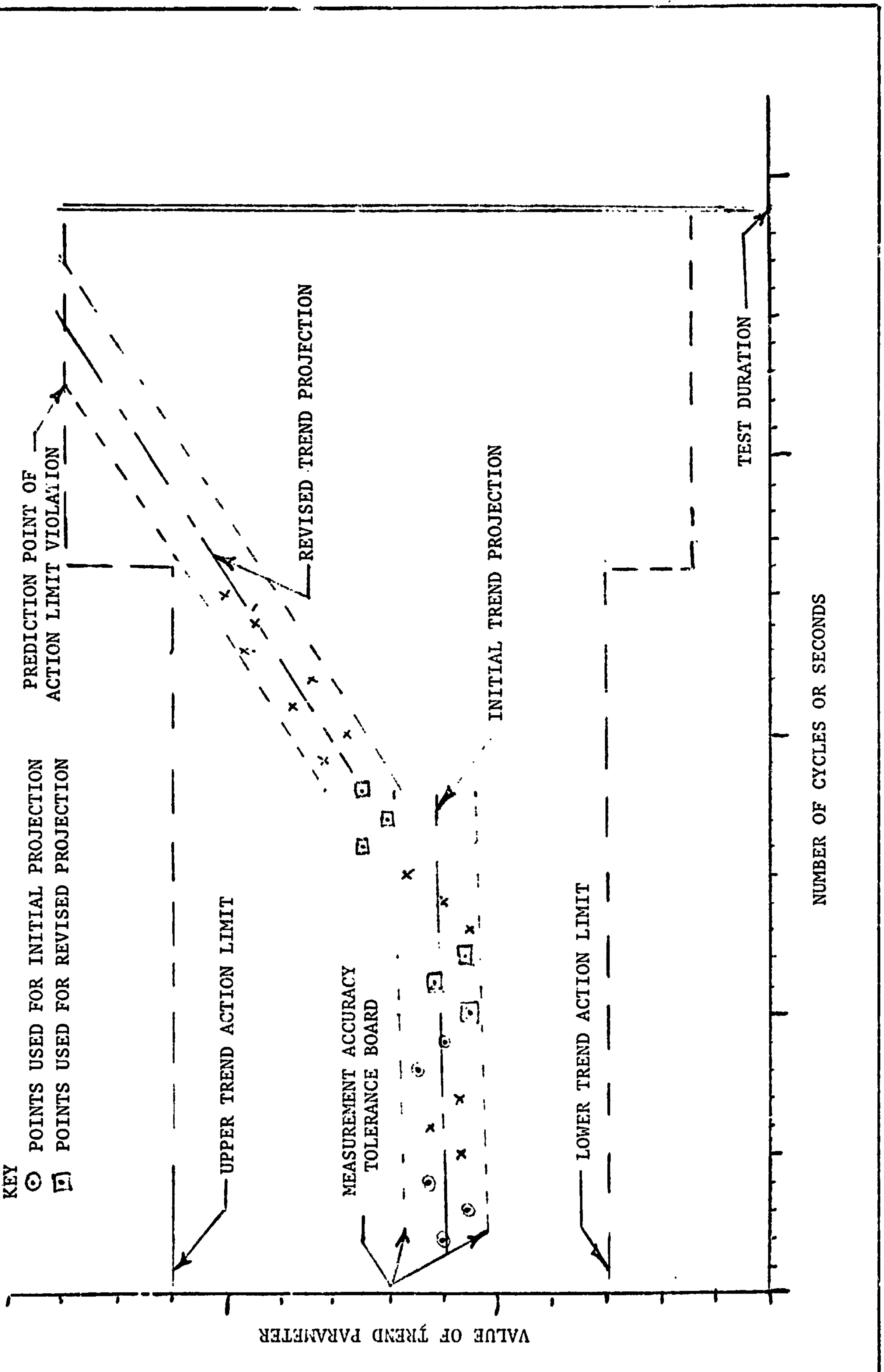
7.7.1 Assuring that trend characteristic monitoring requirements for fabrication inspection or tests are incorporated in appropriate manufacturing/inspection planning. In addition, assuring that detailed test planning documents similarly incorporate specified trend characteristic monitoring requirements.

7.7.2 Assuring that trend data information is properly recorded, analyzed and forwarded as specified.

7.7.3 Assuring that instances of actual or predicted trend action limit violation during a test are recorded as a departure from a requirement and reported to Engineering or Material Review Boards for discrepancy/failure classification.

FIGURE 1

ILLUSTRATION OF TREND DATA MONITORING DEFINITION



NERVA PROGRAM RELIABILITY PROCEDURE	NUMBER: R101-NRP-508	REVISION
	EFFECTIVE DATE: 21 August 1970	CATEGORY III
ANALYSIS AND VERIFICATION OF TREND CHARACTERISTICS	SUPERSEDES: NUMBER: DATE:	
	APPROVED BY:	

1.0 PURPOSE

1.1 The purpose of this procedure is to describe the process to be followed in summarizing and analyzing wearout and trend data deterioration. The activities covered in this procedure are those occurring subsequent to those involving the initial identification of trend characteristics (desc d in R101-NRP-506) and those involved in developing and operating the systems for monitoring those parameters in a specific test (described in R101-NRP-507). The primary effort covered involves the combining of data from several component or engine tests into a summary form. The use of such summaries to verify the continued need for monitoring the trend characteristics, as well as the choice of trend action limits, trend project techniques, etc., is described.

2.0 APPLICABLE DOCUMENTS

2.1 Data Item R101, NERVA Reliability Program Plan

2.2 R101-NRP-506, Identification and Control of Trend Characteristics

2.3 R101-NRP-507, Development and Implementation of the Trend Characteristic Monitoring Systems

3.0 POLICY

3.1 As part of the reliability program, data collected during trend characteristic monitoring activities in fabrication, acceptance, development, prequalification and qualification tests will be continuously summarized and analyzed to accomplish the following objectives:

3.1.1 Identify components that should be redesigned to reduce the rate of wear and deterioration and improve system reliability.

3.1.2 Verify the need to continue the monitoring of deteriorating trend characteristics that cannot be corrected by redesign.

3.1.3 Identify components for which corrective maintenance is likely to be required during normal service life to achieve required system reliability.

3.1.4 Identify trend characteristics that can be deleted from monitoring due to demonstrated stability for periods in excess of service life requirements.

3.1.5 Verify the adequacy of parameters and techniques used in ground test and flight operation trend monitoring such as sampling frequency, plotting format, action limits.

3.1.6 Verify the adequacy and reliability of measurement instrumentation used to monitor trend characteristics.

3.1.7 Provide estimates of frequency of occurrence of component failures resulting from wear and deterioration.

4.0 DEFINITIONS

4.1 Real time trend characteristic monitoring is the acquisition, plotting, projection, and analysis of trend characteristic data at a rate and in a manner adequate to provide advanced warning of premature wear or deterioration of a component.

5.0 PROCEDURE

5.1 A centralized trend data collection and analysis system shall be established to which all trend data observed in component and engine testing will be channeled. Provision shall be made for storage and retrieval by component or subsystem specification number, trend characteristic number, type and location of test, and component serial identity.

5.2 As additional test information is accumulated, and at appropriate periodic intervals, composite plots of trend characteristic parameters shall be prepared combining trend data for the same characteristic for different serial numbered components and/or engines. Provision shall be made for separating the data of component tests from engine tests as well as distinguishing between significant configuration changes.

5.3 Summarized trend data shall be subjected to appropriate statistical analysis to determine one of the following:

5.3.1 That no significant trend exists that will result in a trend limit being exceeded prior to the end of required service life. Depending on the number of components and/or engines tested, a recommendation may be made to discontinue monitoring of such trend characteristics.

5.3.2 That sufficient trend indications exist to warrant continued monitoring; however, component redesign to meet service life requirements is not warranted.

5.3.3 That there is an unacceptably high probability of a trend action limit being exceeded prior to the end of required service life and that either the component must be redesigned or provision must be made for continued real time monitoring and corrective maintenance.

5.4 Summarized trend data shall also be used to verify the adequacy of various features of the real time trend characteristic monitoring performed in direct support of component, subsystem and engine tests and during flight operations. This includes verification of adequacy of:

5.4.1 frequency of sampling

5.4.2 trend projection technique

5.4.3 estimates of measurement accuracy tolerance

5.5 Where real-time trend characteristic monitoring is required, as in engine ground tests and flight operations, summarized trend data anslysis shall be used to evaluate the relationship between the rates of trend deterioration and the protection afforded by the trend action limits. Some cases of rapid deterioration may warrant the modification of trend action limit magnitudes or shifting to faster sampling rates to provide more advanced warning time.

5.6 Summarized trend characteristic data shall also be used to evaluate the adequacy of the measurement instrumentation used to acquire the data, as well as the computational techniques used in its analysis and display. The accuracy in predicting time to action limit violation for use in flight mission success calculation shall also be evaluated.

5.7 Results will be summarized periodically in reports assembled at significant program milestones. Requirements for trend characteristic monitoring during flight operations will be continuously reviewed in terms of related requirement of malfunction detection and fault isolation.

6.0 APPLICABILITY

This procedure is applicable to all approved trend characteristics identified per R101-NRP-506.

7.0 RESPONSIBILITY

7.1 Reliability shall be responsible for:

7.1.1 Collecting, analyzing, and summarizing trend characteristic data as part of the operation of the Reliability Data Center.

7.1.2 Preparing recommendations for:

7.1.2.1 Terminating the monitoring of a trend characteristic

7.1.2.2 Continuing monitoring of a trend characteristic

7.1.2.3 Redesigning a component to eliminate trend characteristics evidencing premature wearout

7.1.2.4 Development of requirements for in-flight trend monitoring, fault isolation, and probability of mission success calculation

7.1.2.5 Improving trend monitoring instrumentation and analytical techniques

7.1.3 Calculating frequency of occurrence of component failure due to wear and deterioration in normal service life.

7.1.4 Determining time or cycles to onset of wearout or deterioration.

7.1.5 Preparing summary inputs for periodic reports and final recommendation for the flight trend monitoring system at CDR.

7.2 Component Design Engineering shall be responsible for:

7.2.1 Analyzing trend characteristic summary data.

7.2.2 Reviewing, approving and acting upon recommendations for terminating trend characteristic monitoring, continuing trend characteristic monitoring, and redesigning to eliminate prematurely trending characteristics.

7.3 Controls and Instrumentation shall be responsible for:

7.3.1 Implementing required changes in trend characteristic monitoring instrumentation and trend data processing equipment in the engine Instrumentation and Control System. Observed results relative to rate of trend deterioration shall be used in evaluation of malfunction detection system requirements.

7.4 Computer Services shall be responsible for:

7.4.1 Developing the computer system for storing, collating, retrieving and summarizing trend data in support of the Reliability Data Center.

7.5 Engine Design shall:

7.5.1 Review trend data results and requirements in terms of related requirements of malfunction detection and fault isolation.

7.5.2 Incorporate summary results of trends data analysis in determining logistics requirements for corrective maintenance.